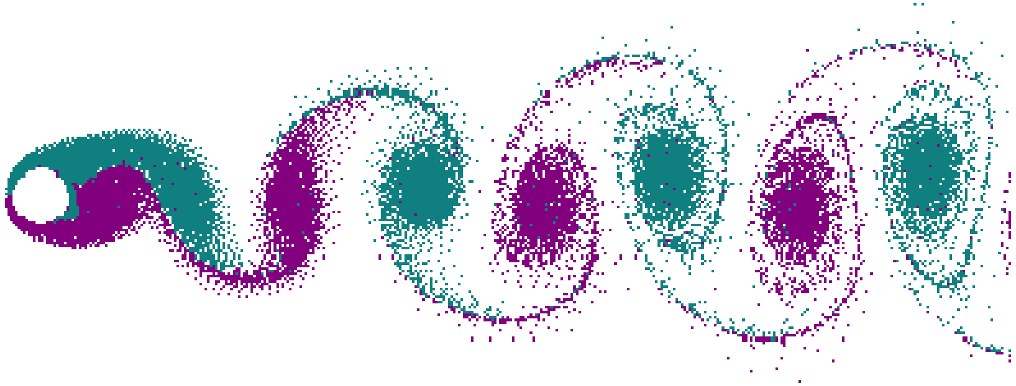
流体

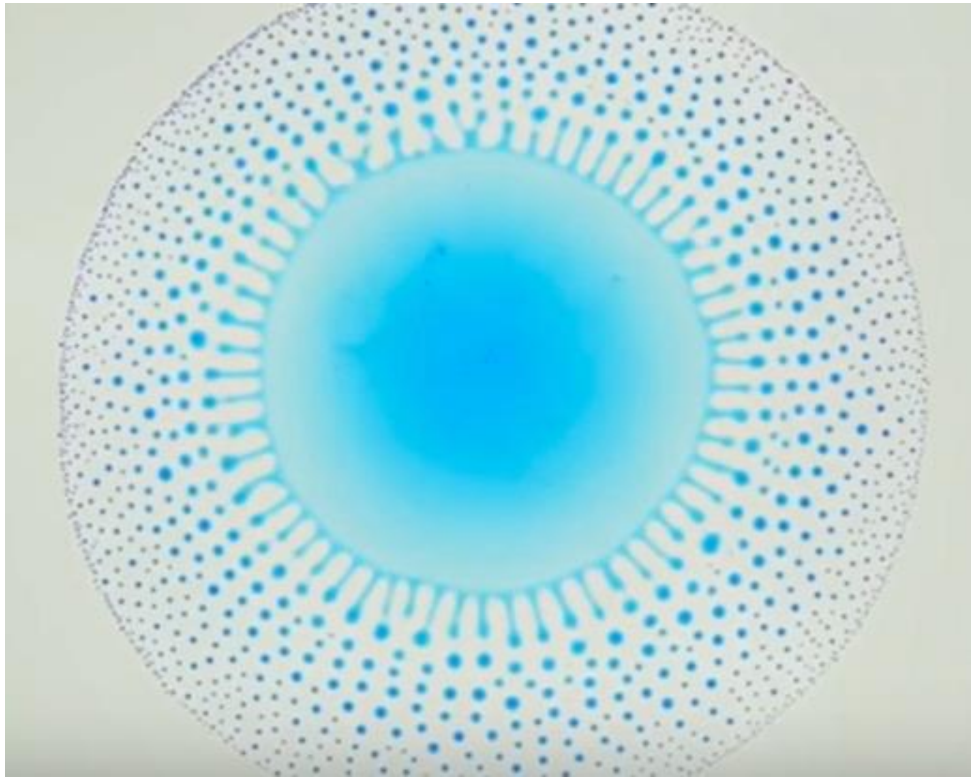
谁用烟云般的字体，在南方的群星间写下你的名字

——聂鲁达

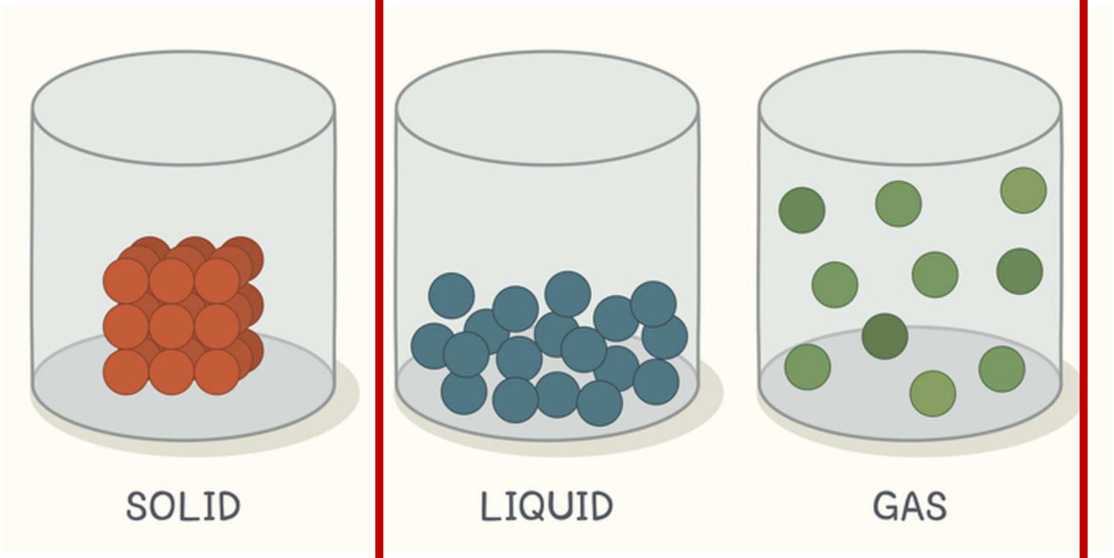
https://www.bilibili.com/video/av34766855?p=33
https://www.bilibili.com/video/av67000894/?spm_id_from=333.788.videocard.0

卡门步伐(Kármán gait)





流体



流体：液体，气体

有相互作用　　弱相互作用

流体的一般性质：

保有空间平移对称性（与固体不同）

连续介质，受力会形变

可产生法向应力（压力），来源与效果：

分子间的相互作用，压缩性弱

分子运动对表面的碰撞，压缩性强

流动性，什么是流动性？

切向应力的消失
- 流体在静止时形变不产生切向应力
- 运动状态下流体有切向应力，粘滞力

其他受力：

重力，磁力，...

流体

如何描述流体？

问题的尺度：

分子

远大于分子尺度：忽略微观的分子运动
远小于宏观尺度：仍然是微元

液滴

运动的描述：

拉格朗日：观察描述同一微元的运动位置，速度

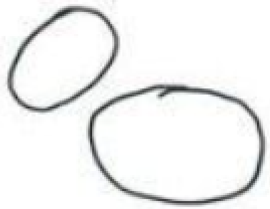
欧拉：固定观察的位置，描述流过该位置的微元

在一定近似下的数学描述(Navier-Stokes, 1850)：

$$\rho(\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=-\nabla p+\nabla\cdot\left\{\mu\left(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{\mathrm{T}}-\frac{2}{3}(\nabla\cdot\boldsymbol{u})\mathbf{I}\right)+\zeta(\nabla\cdot\boldsymbol{u})\mathbf{I}\right\}+\rho\boldsymbol{g}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0$$

找到主体



厘清结构

加上细节

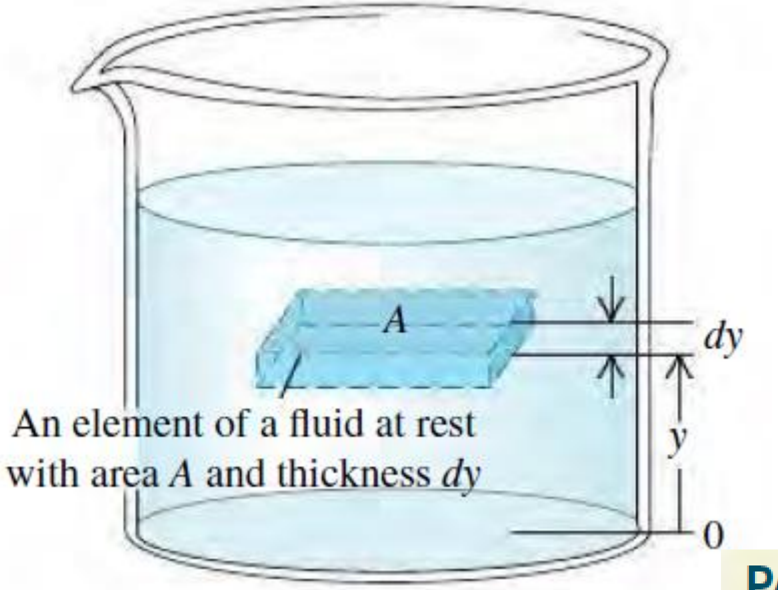
静止流体：

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{u}\cdot\nabla\right)\boldsymbol{u} = -\nabla p + \nabla\cdot\left\{\mu\left(\nabla\boldsymbol{u}+\nabla\boldsymbol{u}^{\mathrm{T}}-\frac{2}{3}(\nabla\cdot\boldsymbol{u})\mathbf{I}\right)+\zeta(\nabla\cdot\boldsymbol{u})\mathbf{I}\right\} + \rho\boldsymbol{g}$$

主要物理量：压强（流体内部的应力）

流体微元的分析 $\Sigma F_y = 0$ so $pA - (p + dp)A - \rho g A\,dy = 0$ 牛顿定律

$$\frac{dp}{dy} = -\rho g$$



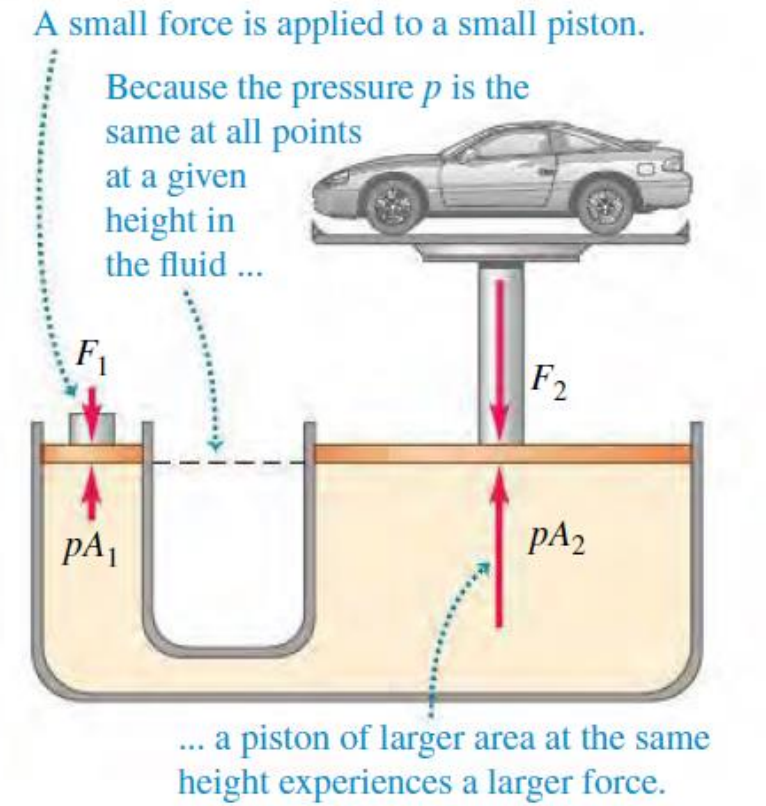
Pressure at depth *h* in a fluid of uniform density → $p = p_0 + \rho g h$

- Uniform density of fluid: ρ
- Depth below surface: h
- Pressure at surface of fluid: p_0
- Acceleration due to gravity ($g > 0$): g

PASCAL'S LAW: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

压强

12.7 The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.



做功：

$$F_1 = pA_1 \qquad F_2 = pA_2$$

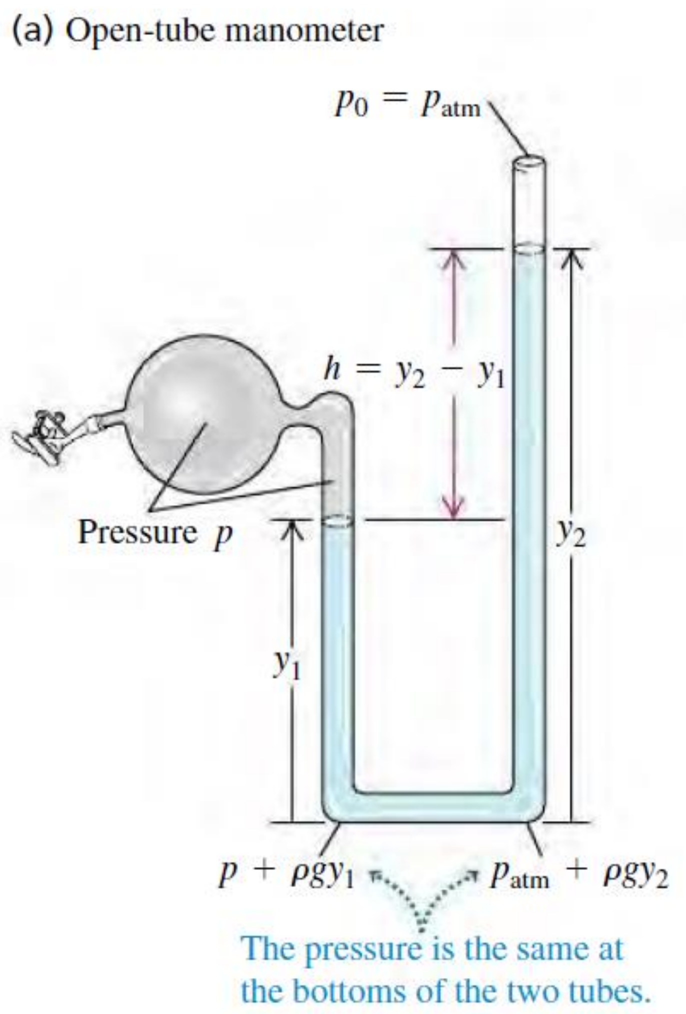
$$ds_1 = dv/A_1 \qquad ds_2 = dv/A_2$$

$$F_1 ds_1 = F_2 ds_2$$

瞬时做功一样，力按比例减小，功程按比例增加。

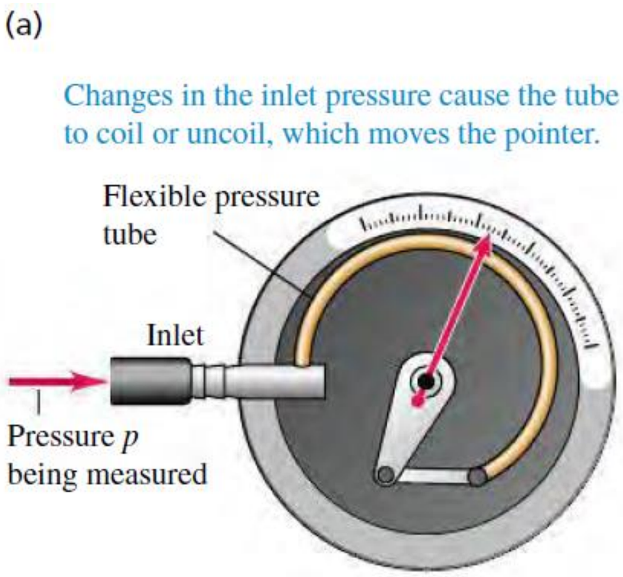
当两边液面高度不相等时？

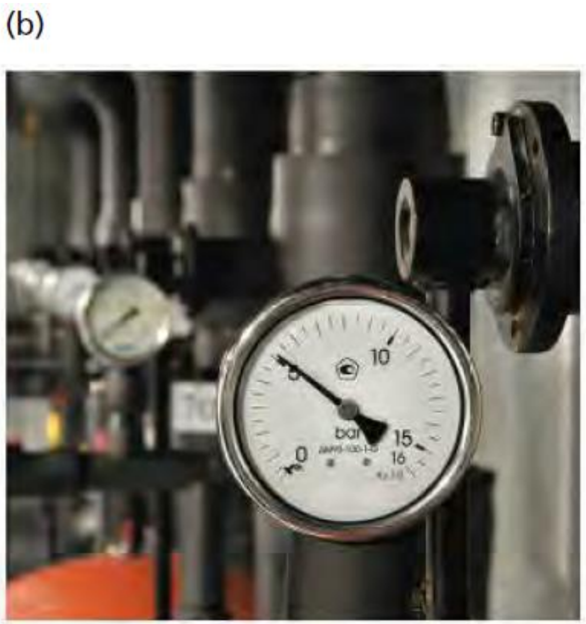
压强计



$$p + \rho g y_1 = p_{atm} + \rho g y_2$$

$$p - p_{atm} = \rho g (y_2 - y_1) = \rho g h$$





阿基米德定律

ARCHIMEDES'S PRINCIPLE: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

12.17 How does the scale reading change when the statue is immersed in water?

$$h_1 A = V_0$$

$$h_2 A = V_0 + V_g$$

液体高度差

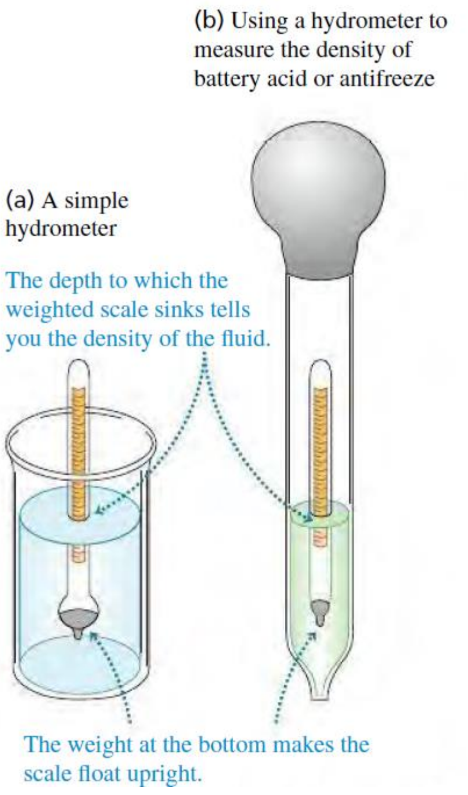
$$\rho g(h_2 - h_1)A = \Delta F_s$$

压力差

$$\rho g V_g = \Delta F_s$$

V_0可不可以大于V_g？

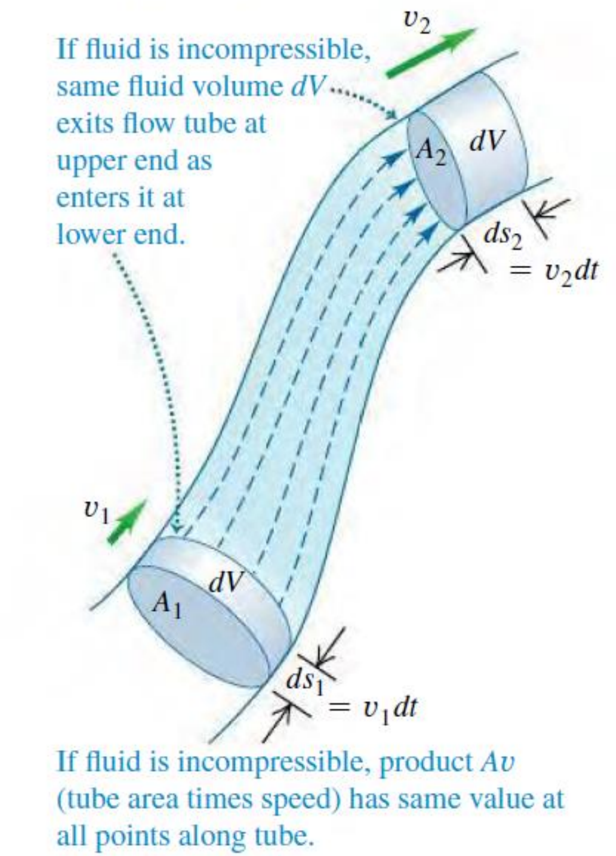
12.12 Measuring the density of a fluid.



最简单的流动模型：理想流体

质量守恒：$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{u})=0$

12.21 A flow tube with changing cross-sectional area.



假设：理想流体不可压缩 ➡ 密度均匀、不变

$$\rho=\text{const.},\ \frac{\partial}{\partial t}\rho=0$$

对于固定的体积，流出的质量=流进的质量

连续性定理 $\rho A_1v_1\,dt=\rho A_2v_2\,dt$

Continuity equation for an incompressible fluid

$$A_1v_1=A_2v_2$$

Cross-sectional area of flow tube at two points (see Fig. 12.21)

Speed of flow at the two points

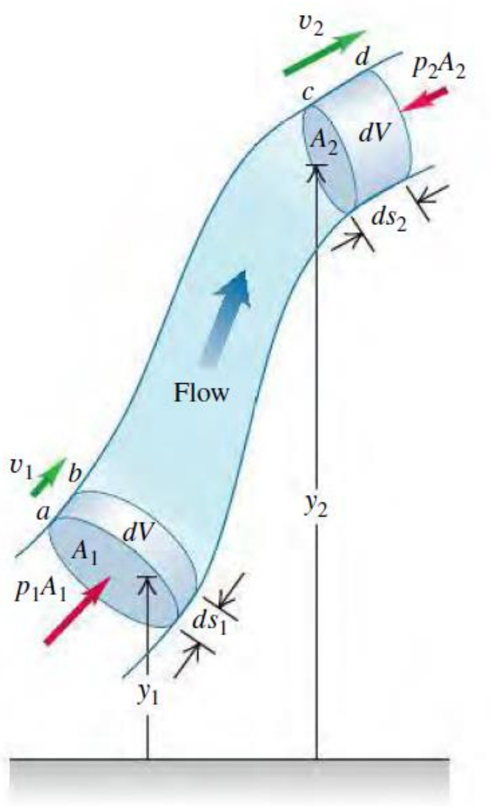
Volume flow rate of a fluid

$$\frac{dV}{dt}=Av$$

Cross-sectional area of flow tube

Speed of flow

Bernoulli’s equation $\rho(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \nabla\cdot\left\{\mu\left(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^{\mathrm{T}} - \frac{2}{3}(\nabla\cdot\boldsymbol{u})\mathbf{I}\right) + \zeta(\nabla\cdot\boldsymbol{u})\mathbf{I}\right\} + \rho\boldsymbol{g}$



假设：不可压缩 + 无粘滞、稳定流

$dW = p_1A_1\,ds_1 - p_2A_2\,ds_2 = (p_1 - p_2)dV$ 压力做功

能量守恒

转化为动量 $dK = \frac{1}{2}\rho\,dV(v_2^2 - v_1^2)$ 和势能 $dU = \rho\,dV\,g(y_2 - y_1)$

$$(p_1 - p_2)dV = \frac{1}{2}\rho\,dV(v_2^2 - v_1^2) + \rho\,dV\,g(y_2 - y_1)$$

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

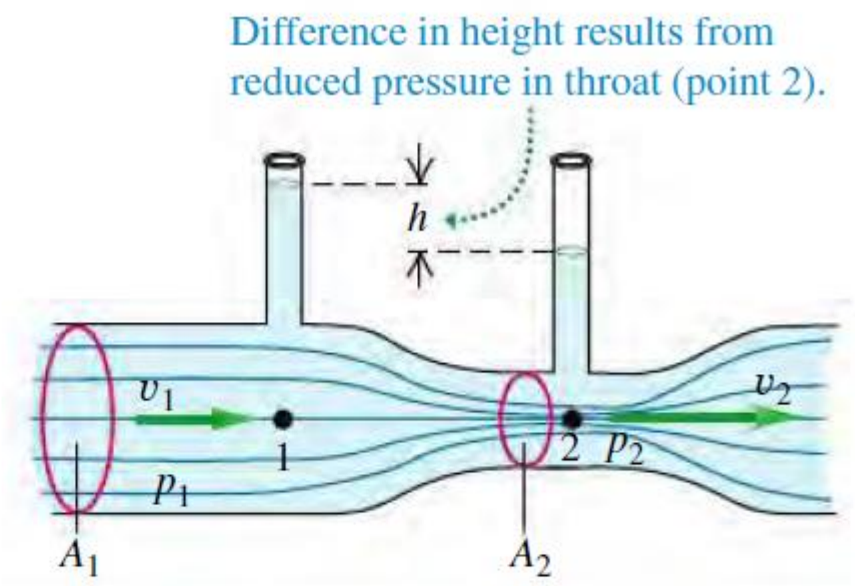
Bernoulli’s equation for an ideal, incompressible fluid:

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

Pressure; Fluid density; Value is same at all points in flow tube.; Acceleration due to gravity; Elevation; Flow speed

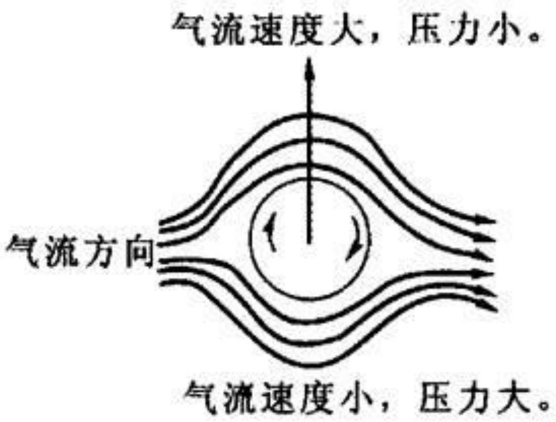
Bernoulli's equation应用

文氏管流量计

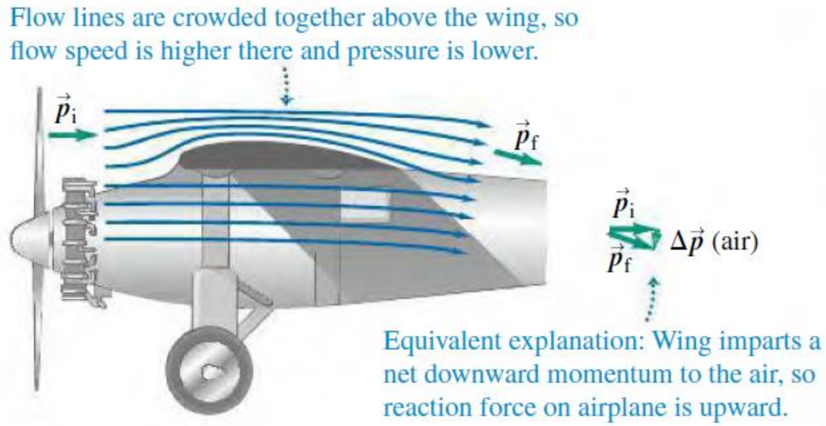


$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

香蕉球



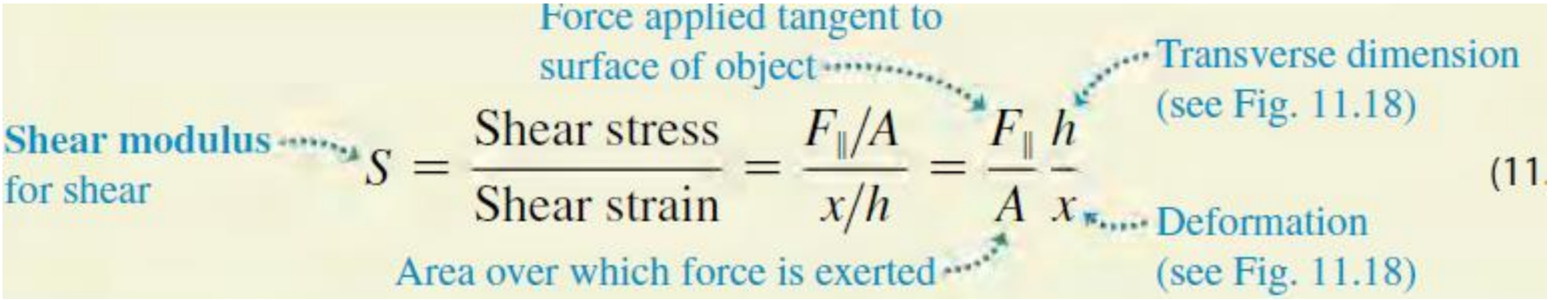
机翼抬升



切向应力

切向应力是黏性的来源

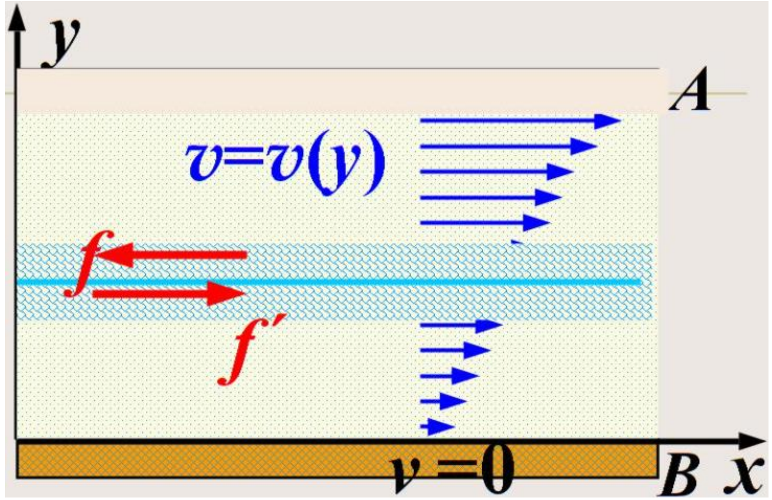
仅在流动时出现，是阻碍液体流动的力（与摩擦力相似）



对于流体，应力与速度梯度相关，$\frac{dv}{dy}$

物理图像：不同速度的流层间存在“摩擦力”

这个力试图消除速度梯度



牛顿流体

对照切向应力

Force applied tangent to surface of object

Transverse dimension (see Fig. 11.18)

Shear modulus for shear

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A}\frac{h}{x} \quad (11.$$

Deformation (see Fig. 11.18)

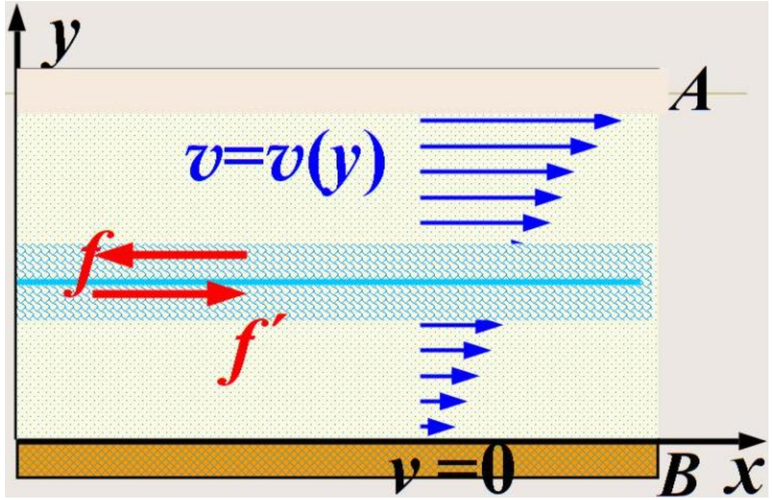
Area over which force is exerted

粘滞力与速度梯度相关： $\tau = \frac{f}{A} = \mu \frac{dv}{dy}$

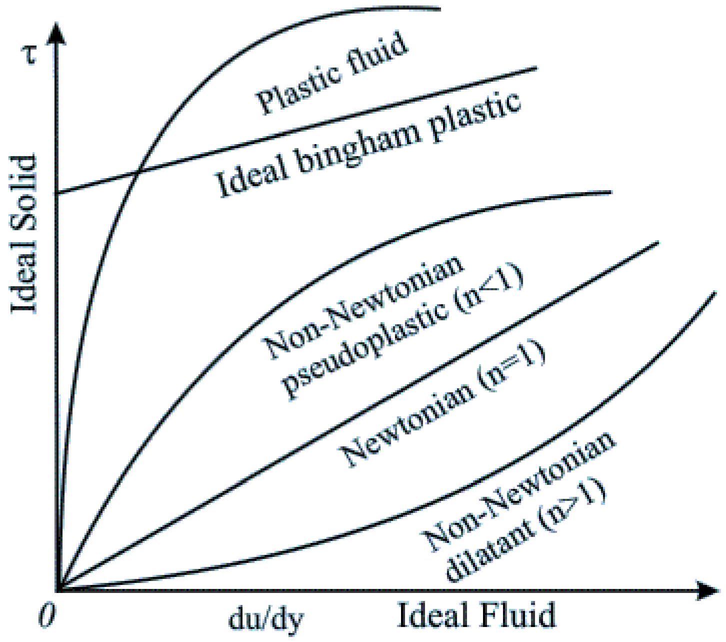
粘滞系数：μ, 注意量纲与模量S不同

牛顿流体： μ是常数的流体

温度上升时， μ下降， μ上升　　为什么？



非牛顿流体



n: flow behaviour index

- 任何偏离线性的流体，都被称为非牛顿流体。
- 等量du/dy时，τ越大，越类似固体；τ=0，理想液体。

1. Dilatant - 胀流性非牛顿液体，玉米淀粉+水
2. Pseudoplastic -假塑性非牛顿液体，果酱；高分子；。。。
3. Bingham plastic – 固流体，蛋黄酱，花生酱。。。

Wetting and interface