

流体

谁用烟云般的字体，在南方的群星间写下你的名字

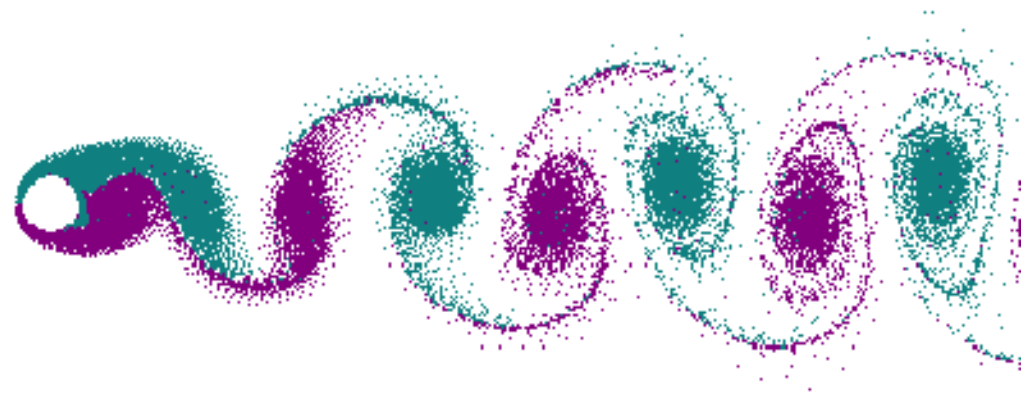
——聂鲁达

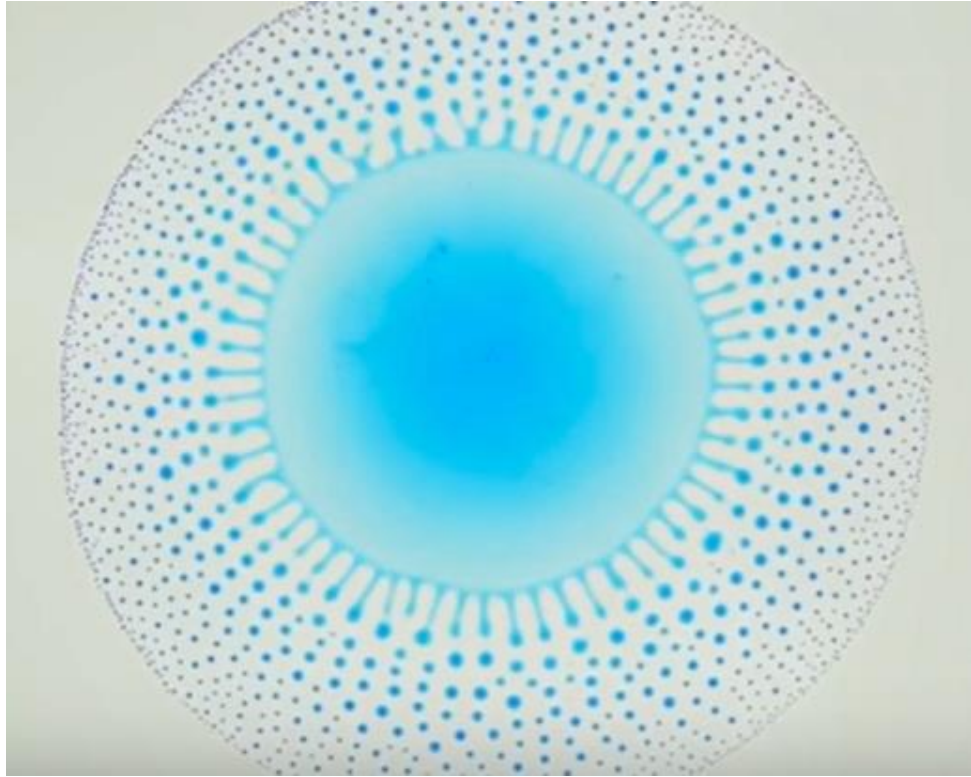
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卡门步伐(Kármán gait)





流体

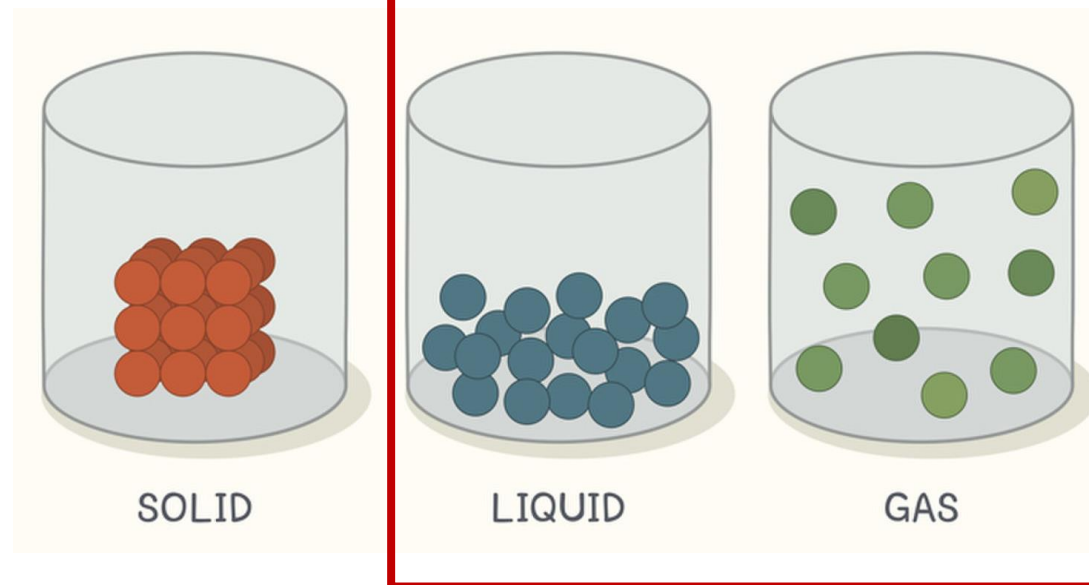
流体：液体，气体
有相互作用 弱相互作用

流体的一般性质：

保有空间平移对称性（与固体不同）
连续介质，受力会形变
可产生法向应力（压力），来源与效果：

流动性，什么是流动性？

切向应力的消失 { 流体在静止时形变不产生切向应力
运动状态下流体有切向应力，粘滞力



分子间的相互作用，压缩性弱

分子运动对表面的碰撞，压缩性强

其他受力：

重力，磁力，...

流体

如何描述流体？

分子

问题的尺度：

远大于分子尺度：忽略微观的分子运动
远小于宏观尺度：仍然是微元

液滴

运动的描述：

拉格朗日：观察描述同一微元的运动位置，速度

欧拉：固定观察的位置，描述流过该位置的微元

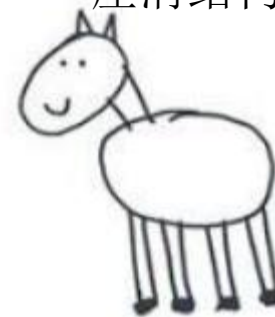
在一定近似下的数学描述(Navier-Stokes, 1850):

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla p + \nabla \cdot \left\{ \mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \zeta (\nabla \cdot \mathbf{u}) \mathbf{I} \right\} + \rho \mathbf{g}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

找到主体



厘清结构



加上细节



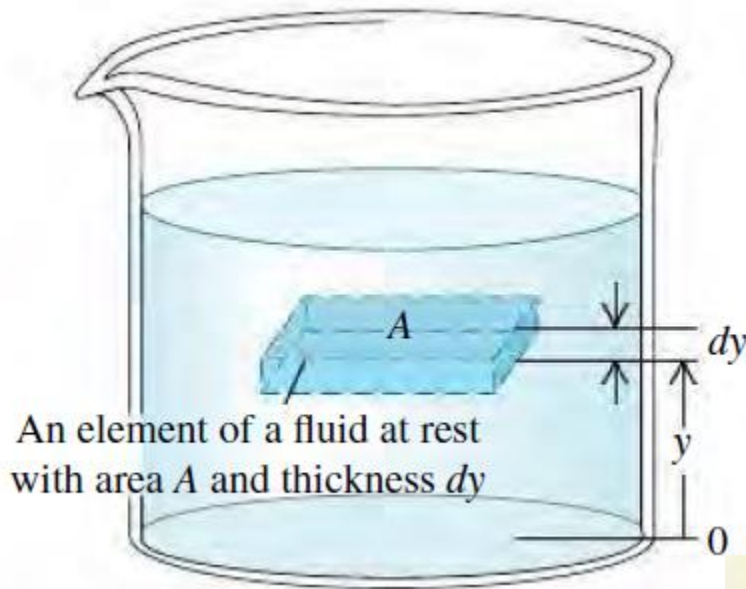
静止流体:

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla p + \nabla \cdot \left\{ \mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \zeta (\nabla \cdot \mathbf{u}) \mathbf{I} \right\} + \rho \mathbf{g}$$

主要物理量: 压强 (流体内部的应力)

流体微元的分析 $\Sigma F_y = 0$ so $pA - (p + dp)A - \rho g A dy = 0$ 牛顿定律

$$\frac{dp}{dy} = -\rho g$$



Pressure at depth h
in a fluid of uniform
density

$$p = p_0 + \rho g h$$

Pressure at surface of fluid

Uniform density of fluid

Depth below surface

Acceleration due to gravity ($g > 0$)

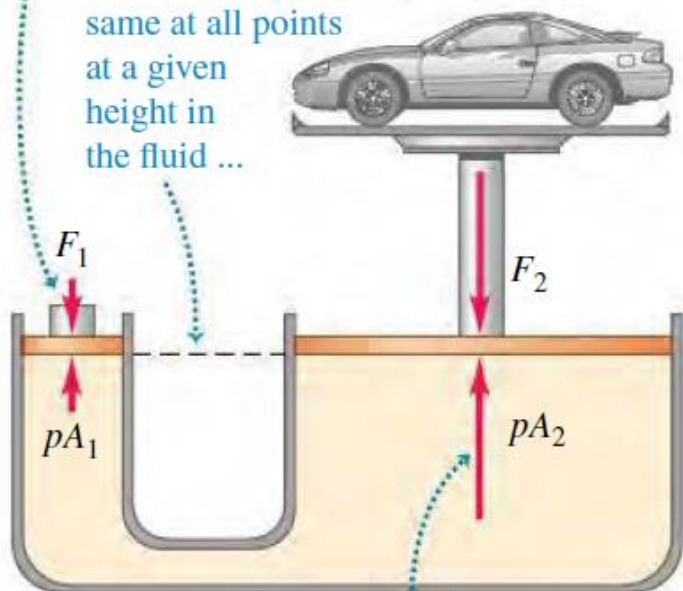
PASCAL'S LAW: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.

压强

12.7 The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.

A small force is applied to a small piston.

Because the pressure p is the same at all points at a given height in the fluid ...



... a piston of larger area at the same height experiences a larger force.

做功:

$$F_1 = pA_1 \quad F_2 = pA_2$$

$$ds_1 = dv/A_1 \quad ds_2 = dv/A_2$$

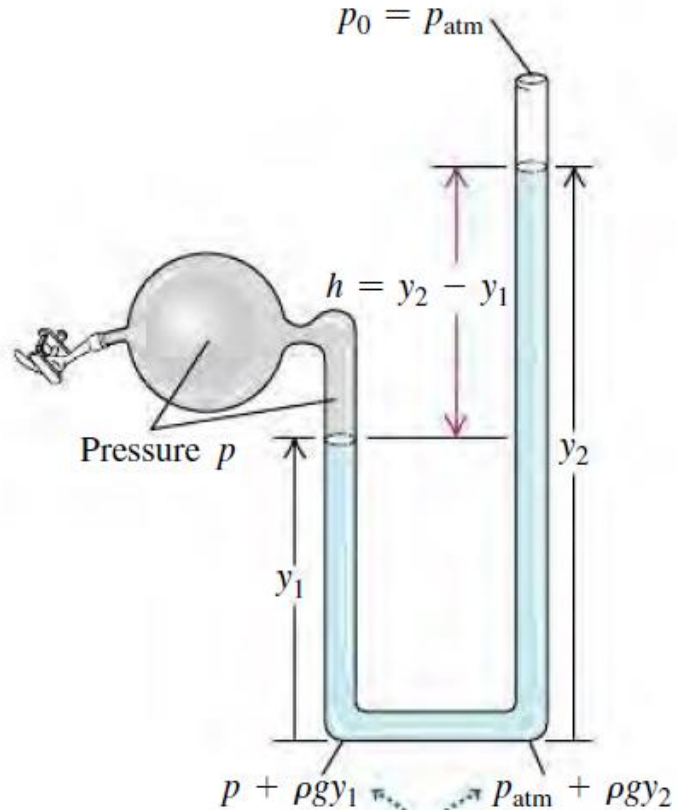
$$F_1 ds_1 = F_2 ds_2$$

瞬时做功一样，力按比例减小，功程按比例增加。

当两边液面高度不相等时？

压强计

(a) Open-tube manometer



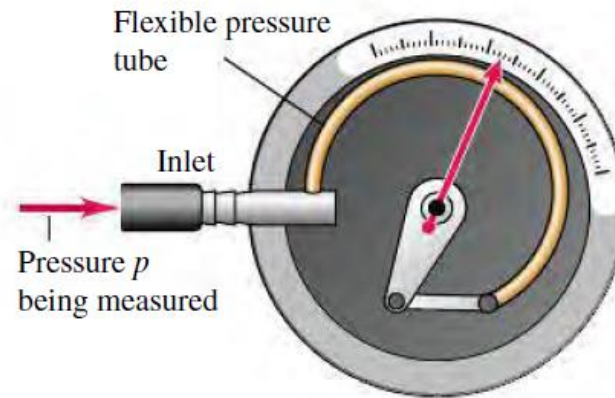
The pressure is the same at the bottoms of the two tubes.

$$p + \rho g y_1 = p_{\text{atm}} + \rho g y_2$$

$$p - p_{\text{atm}} = \rho g (y_2 - y_1) = \rho g h$$

(a)

Changes in the inlet pressure cause the tube to coil or uncoil, which moves the pointer.



(b)



阿基米德定律

ARCHIMEDES'S PRINCIPLE: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

12.17 How does the scale reading change when the statue is immersed in water?



$$h_1 A = V_0$$

液体高度差

$$h_2 A = V_0 + V_g$$

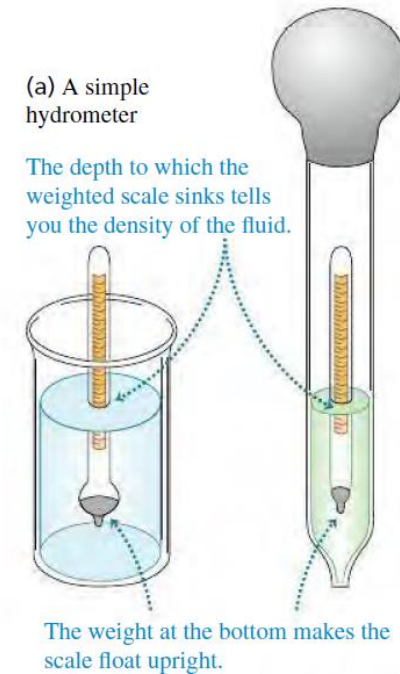
$$\rho g (h_2 - h_1) A = \Delta F_s \quad \text{压力差}$$

$$\rho g V_g = \Delta F_s$$

V_0 可不可以大于 V_g ?

12.12 Measuring the density of a fluid.

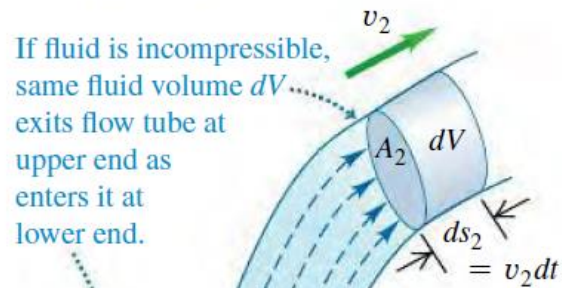
(b) Using a hydrometer to measure the density of battery acid or antifreeze



最简单的流动模型：理想流体

质量守恒： $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$

12.21 A flow tube with changing cross-sectional area.



If fluid is incompressible, same fluid volume dV exits flow tube at upper end as enters it at lower end.

If fluid is incompressible, product Av (tube area times speed) has same value at all points along tube.

假设：理想流体不可压缩 → 密度均匀、不变
 $\rho = \text{const.}, \frac{\partial}{\partial t} \rho = 0$

对于固定的体积，流出的质量=流进的质量

连续性定理 $\rho A_1 v_1 dt = \rho A_2 v_2 dt$

Continuity equation for an incompressible fluid $A_1 v_1 = A_2 v_2$

Cross-sectional area of flow tube at two points (see Fig. 12.21)

Speed of flow at the two points

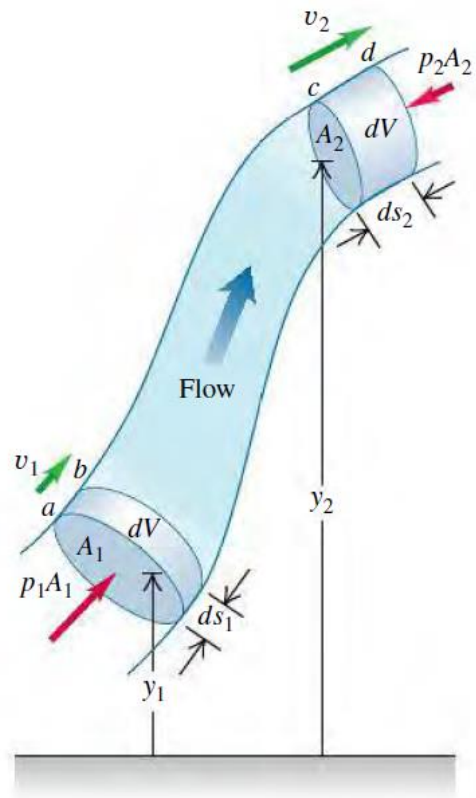
Volume flow rate of a fluid $\frac{dV}{dt} = Av$

Cross-sectional area of flow tube

Speed of flow

Bernoulli's equation $\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla p + \nabla \cdot \left\{ \mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \right) + \zeta (\nabla \cdot \mathbf{u}) \mathbf{I} \right\} + \rho \mathbf{g}$

假设：不可压缩 + 无粘滞、稳定流



能量守恒

$$dW = p_1 A_1 ds_1 - p_2 A_2 ds_2 = (p_1 - p_2) dV \quad \text{压力做功}$$

$$\text{转化为动量 } dK = \frac{1}{2} \rho dV (v_2^2 - v_1^2) \text{ 和势能 } dU = \rho dV g (y_2 - y_1)$$

$$(p_1 - p_2) dV = \frac{1}{2} \rho dV (v_2^2 - v_1^2) + \rho dV g (y_2 - y_1)$$

$$p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho g (y_2 - y_1)$$

$$p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Bernoulli's equation for an ideal, incompressible fluid:

$$p + \rho g y + \frac{1}{2} \rho v^2 = \text{constant}$$

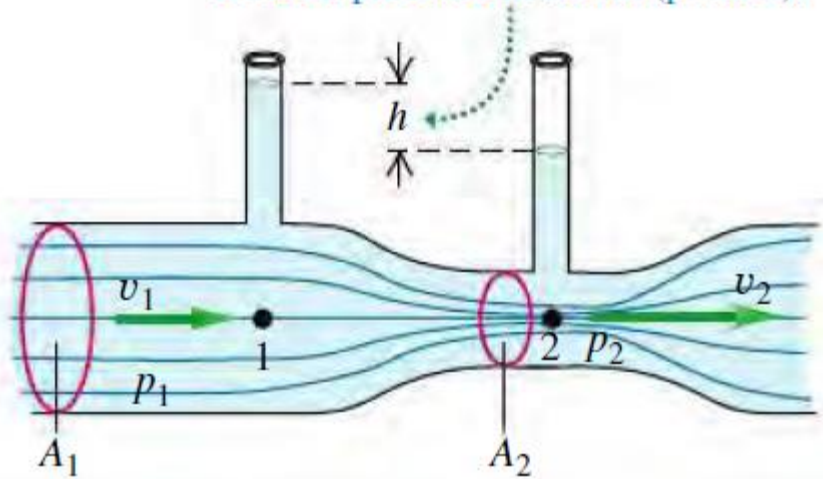
Pressure Fluid density Value is same at all points in flow tube.

Acceleration due to gravity Elevation Flow speed

Bernoulli's equation应用

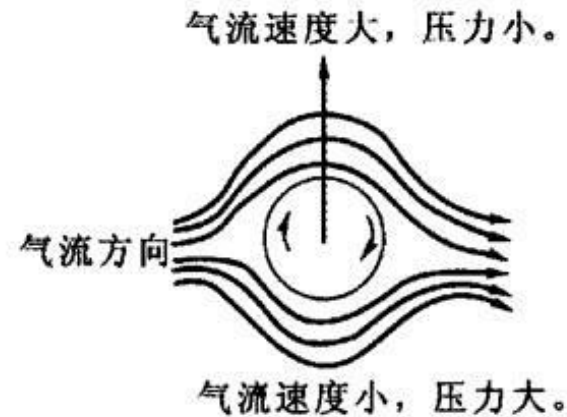
文氏管流量计

Difference in height results from reduced pressure in throat (point 2).



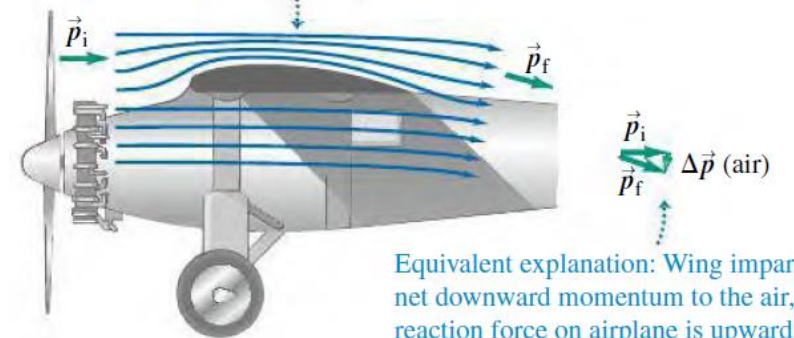
$$v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$$

香蕉球



机翼抬升

Flow lines are crowded together above the wing, so flow speed is higher there and pressure is lower.



Equivalent explanation: Wing imparts a net downward momentum to the air, so reaction force on airplane is upward.

切向应力

切向应力是黏性的来源

仅在流动时出现，是阻碍液体流动的力（与摩擦力相似）

$$\text{Shear modulus for shear } S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel} h}{A x} \quad (11.1)$$

Force applied tangent to surface of object

Transverse dimension (see Fig. 11.18)

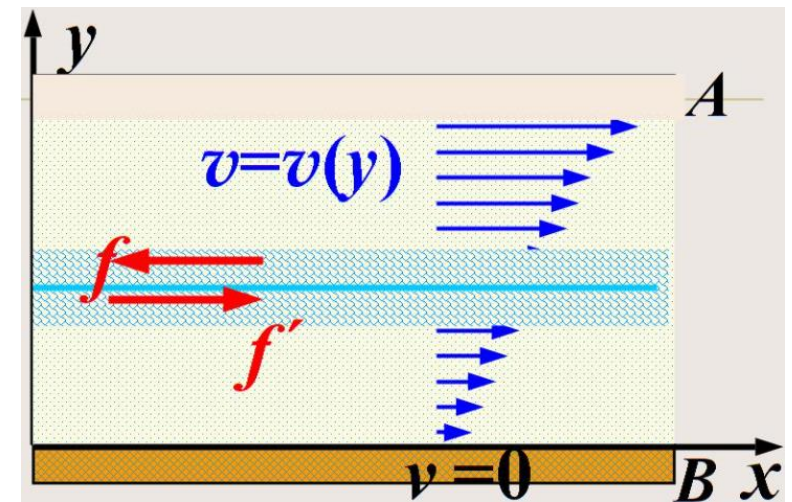
Deformation (see Fig. 11.18)

Area over which force is exerted



对于流体，应力与速度梯度相关， $\frac{dv}{dy}$

物理图像：不同速度的流层间存在“摩擦力”
这个力试图消除速度梯度



牛顿流体

对照切向应力

$$\text{Shear modulus for shear } S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel} h}{A x} \quad (11.1)$$

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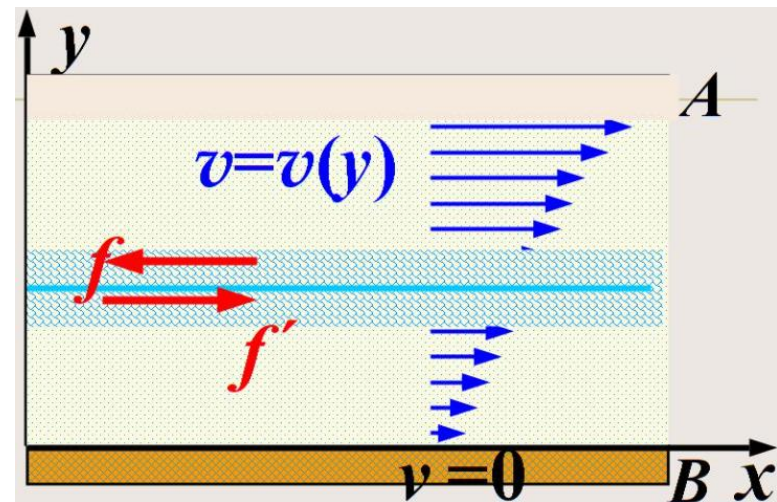
Deformation (see Fig. 11.18)

粘滞力与速度梯度相关: $\tau = \frac{f}{A} = \mu \frac{dv}{dy}$

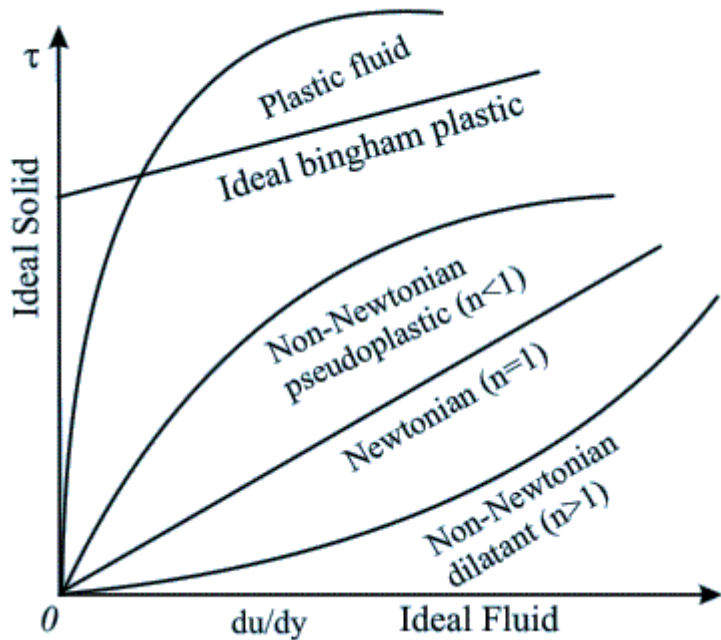
粘滞系数: μ , 注意量纲与模量S不同

牛顿流体: μ 是常数的流体

温度上升时, μ 下降, μ 上升 为什么?



非牛顿流体



n: flow behaviour index

- 任何偏离线性的流体，都被称为非牛顿流体。
 - 等量 du/dy 时， τ 越大，越类似固体； $\tau=0$ ，理想液体。
1. Dilatant - 胀流性非牛顿液体，玉米淀粉+水
 2. Pseudoplastic - 假塑性非牛顿液体，果酱；高分子；。。。
 3. Bingham plastic - 固流体，蛋黄酱，花生酱。。。

Wetting and interface

