

平衡和弹性

平衡和弹性



凡为将者，当以刚柔相济，不可徒恃其勇。

平衡

质点 -- $\{x, y, z\}$ 三个运动自由度

质点系 - $\{x, y, z\} * n$ 个运动自由度

刚体 - $\{x, y, z\} + \{\alpha, \beta, \gamma\}$ 六个运动自由度

刚体的运动可以分解为质心的平动 + 绕质心的转动

质点的平衡 - 牛顿第一定律：合力为零

刚体的平衡 - 合力为零 + 合力矩为零

$$\sum \tau_i = \frac{dJ}{dt} = 0 \quad \text{匀速旋转?}$$

地表对象的平衡：重力场

$$\sum \vec{F} = \mathbf{0}$$

$$Mg = (m_1 + m_2 + m_3 + \dots)g = \sum m_i g$$

$$\sum \vec{\tau} = \mathbf{0}$$

$$\begin{aligned}\vec{\tau} &= \sum_i \vec{\tau}_i = \vec{r}_1 \times m_1 \vec{g} + \vec{r}_2 \times m_2 \vec{g} + \dots \\ &= \left(\sum_i m_i \vec{r}_i \right) \times \vec{g} = \vec{r}_{\text{cm}} \times M \vec{g}\end{aligned}$$

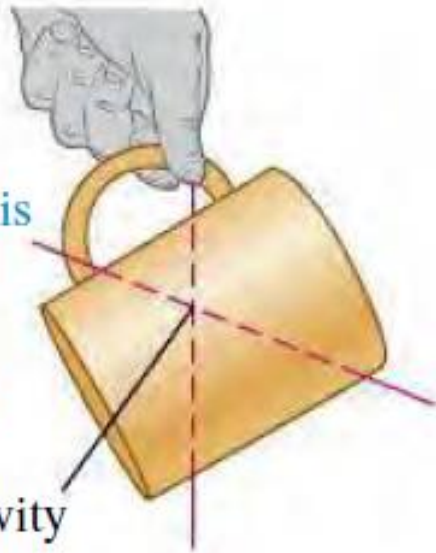
质心 \rightarrow 重心：源于重力仅正比于质量，是一个特殊情况。



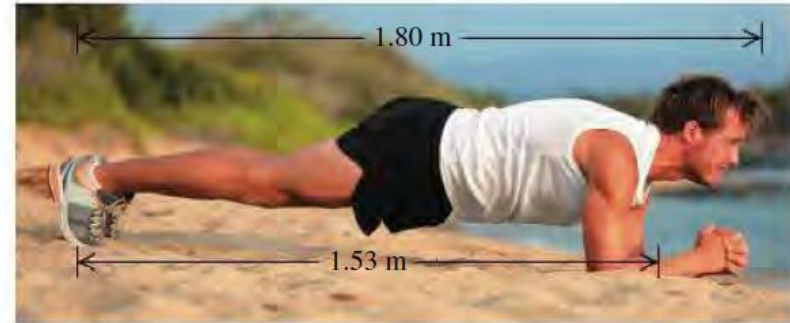
例子

② Now suspend the mug from a different point. A vertical line extending down from this point intersects the first line at the center of gravity (which is inside the mug).

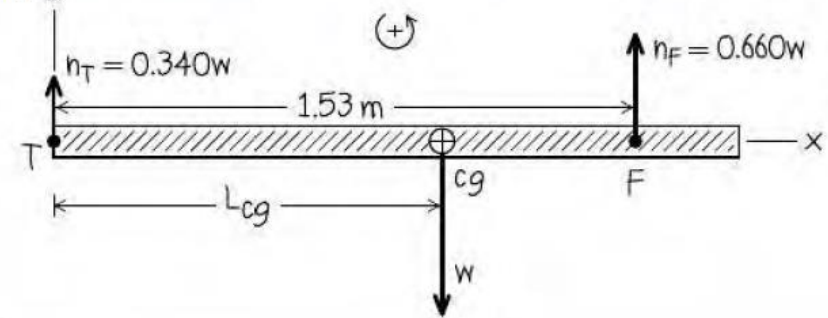
Center of gravity



(a)



(b)

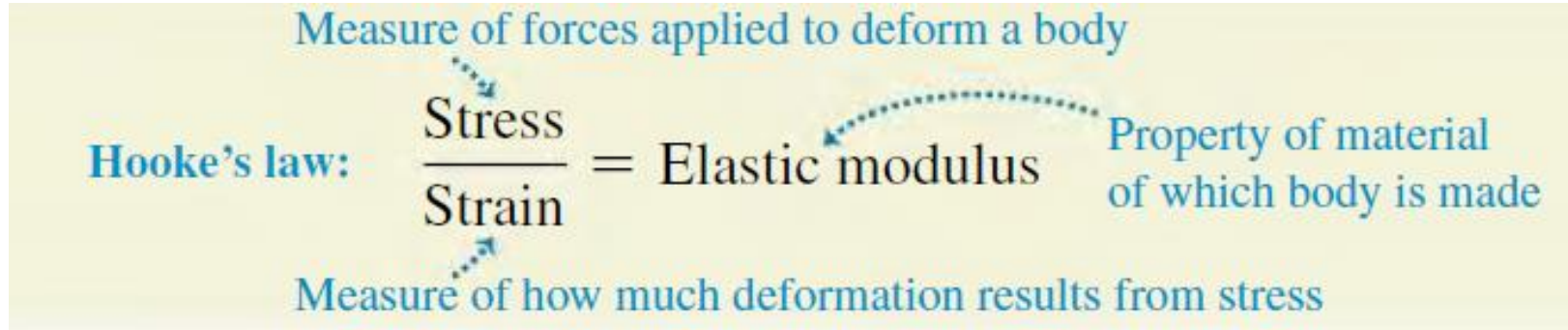


$$\sum \tau_R = 0.340w(0) - wL_{cg} + 0.660w(1.53 \text{ m}) = 0$$
$$L_{cg} = 1.01 \text{ m}$$

例子

圆锥上细绳与小球

应力和应变



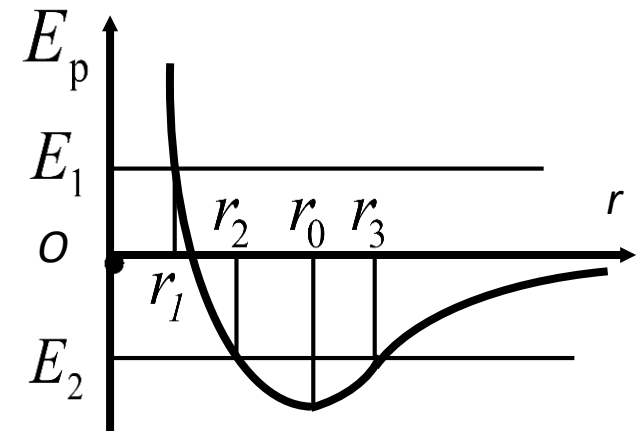
$$E = E(Q_1) \rightarrow E - E_0 = a_0 + a_1 (Q_1 - Q_{10}) + a_2 (Q_1 - Q_{10})^2 + \dots$$

a_0 : 定义为0, $E_0 = E(Q_{10})$

a_1 : 平衡点一阶微分系数为0

第一个重要项: $a_2 : -dE/dQ = F; dF/dQ \sim a_2$

如何更一般地描述 dF/dQ ?



应力和应变

如何更一般地描述 dF/dQ ?

简化模型：考虑全同弹簧的并联与串联，如何修正胡克定律？

$$F = k\Delta l$$



$k, l, \Delta l$

$N = 1$

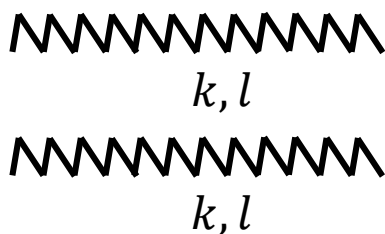


k, l k, l

$$F = k\Delta l = \frac{k}{2}(2\Delta l) = \frac{k}{2}\Delta l_{tot}$$

$$\Delta l_{tot} = 2\Delta l, \quad l_{tot} = 2l$$

$N = 2$

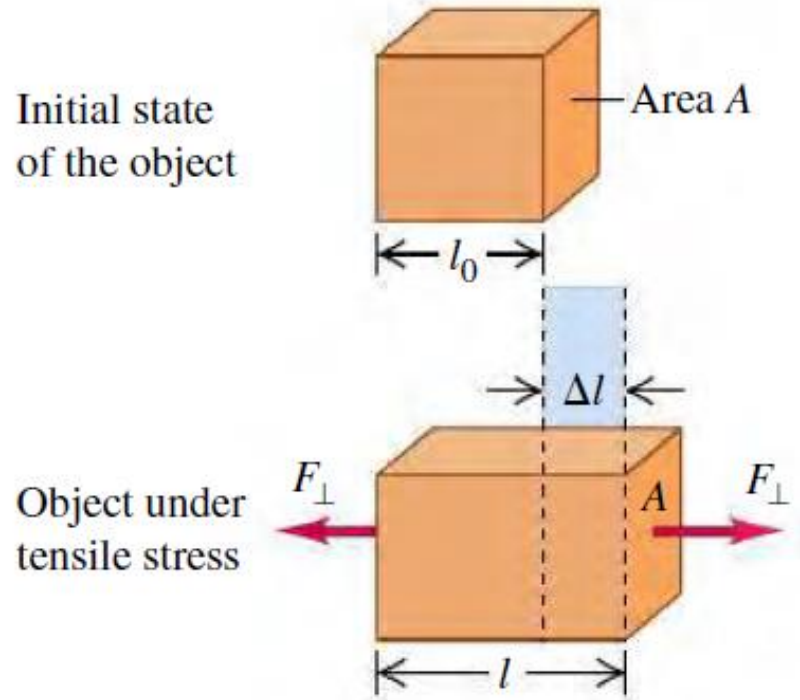


k, l
 k, l

$$F = 2k\Delta l = 2k\Delta l_{tot}$$
$$\Delta l_{tot} = \Delta l, \quad l_{tot} = l$$

$$\frac{F}{N} = kl \frac{\Delta l_{tot}}{l_{tot}}$$

Tensile and Compressive stress and strain



$$\text{Tensile stress} = \frac{F_{\perp}}{A} \quad \text{Tensile strain} = \frac{\Delta l}{l_0}$$

$$\text{Tensile stress} = \frac{F_{\perp}}{A}$$

$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

$$\text{人站在地面上: } 65 \text{ kg} \times 9.8 \text{ N/kg} (8 \times 24 \times 2 \text{ cm}^2) \\ \sim 1.7 \times 10^4 \text{ N/m}^2$$

$$\text{Tensile strain} = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

1m的刚性材料 – 一般“常规受力”弹性形变在十微米 (10^{-5} m)量级

杨氏模量 - Young's modulus

Young's modulus for tension

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_{\perp}/A}{\Delta l/l_0} = \frac{F_{\perp}}{A} \frac{l_0}{\Delta l}$$

Force applied perpendicular to cross section

Original length (see Fig. 11.14)

Elongation (see Fig. 11.14)

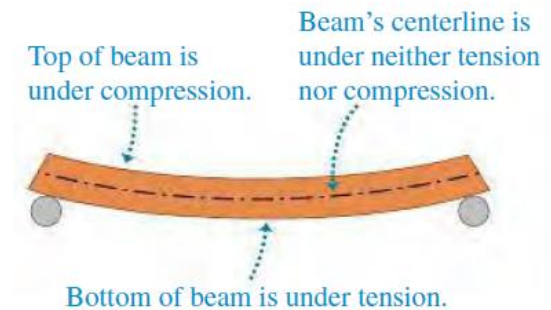
Cross-sectional area of object

TABLE 11.1 Approximate Elastic Moduli

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Silicone rubber	0.001×10^{10}	0.2×10^{10}	0.0002×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}
Tendon (typical)	0.12×10^{10}	—	—

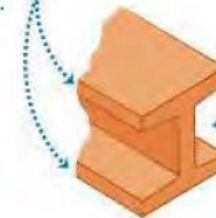
Y越小，材料越容易变形。
Y一样，同样受力，面积越小越容易变形。

(a)



(b)

The top and bottom of an I-beam are broad to minimize the compressive and tensile stresses.



The beam can be narrow near its centerline, which is under neither compression nor tension.

Tensile and Compressive: 拉 vs 压

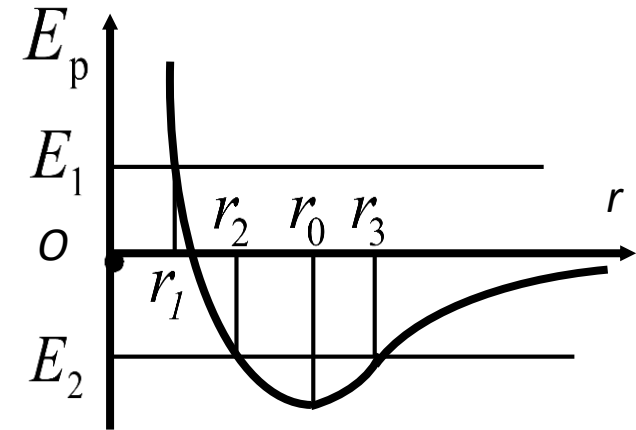
$$E = E(Q_1, Q_2, Q_3, \dots) \rightarrow E - E_0 = a_0 + a_1(Q_1 - Q_{10}) + a_2(Q_1 - Q_{10})^2 + \dots$$

a_0 : 定义为0

a_1 : 平衡点一阶微分系数为0

第一个重要项: $a_2 : -dE/dQ = F; dF/dQ \sim a_2$

如何处理 Q_2, Q_3, \dots ?



相对于平衡点，微分是对称的，拉与压描述相同。

杨氏模量 - 声速 - 声子

我们提到的第一个具体到材料的，宏观，统计的系统参数。

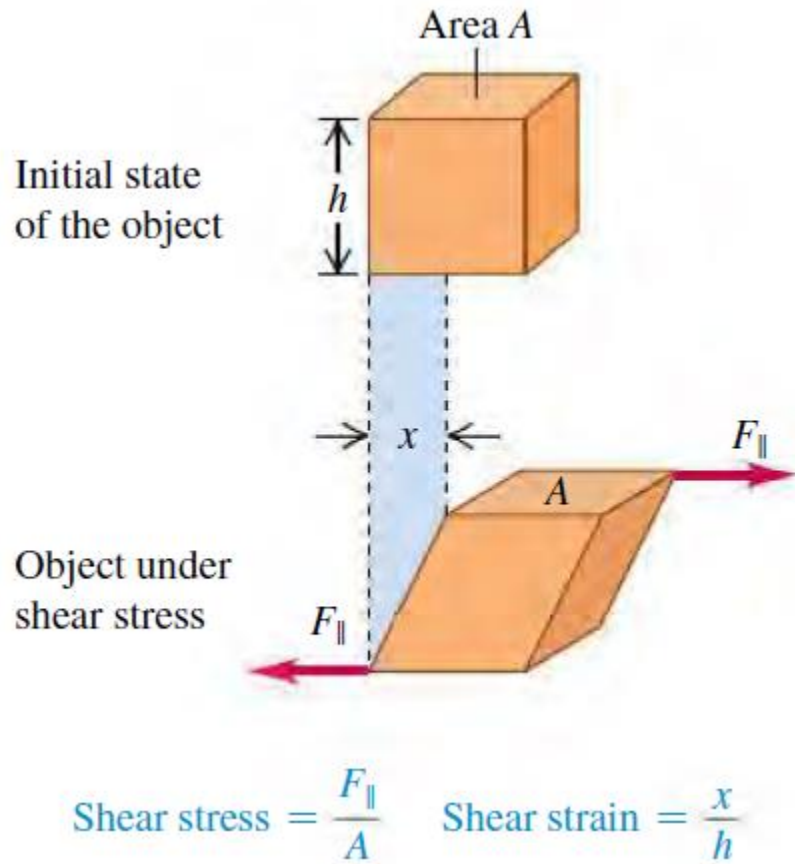


$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

$$c = \sqrt{\frac{Y}{\rho}}$$

微观延伸，量子化 → 声子

Shear Stress and Strain – 剪切应力应变



$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

$$\text{Shear strain} = \frac{x}{h}$$

Force applied tangent to surface of object

Transverse dimension (see Fig. 11.18)

Deformation (see Fig. 11.18)

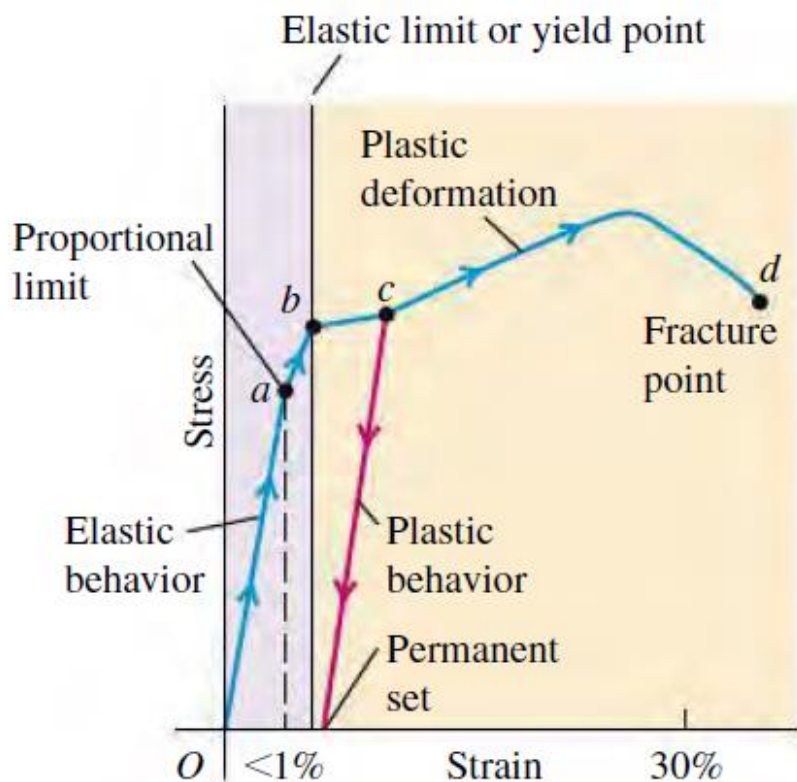
Area over which force is exerted

Shear modulus for shear

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A} \frac{h}{x} \quad (11.18)$$

弹性和塑性

11.19 Typical stress-strain diagram for a ductile metal under tension.



弹性形变：可恢复

塑性形变：不可恢复

TABLE 11.3 Approximate Breaking Stresses

Material	Breaking Stress (Pa or N/m ²)
Aluminum	2.2×10^8
Brass	4.7×10^8
Glass	10×10^8
Iron	3.0×10^8
Steel	$5-20 \times 10^8$
Tendon (typical)	1×10^8