EE160 2025

A Quick Review

1线性系统：定义，满足的两个条件

- 叠加原理
- 齐次性

Linear System

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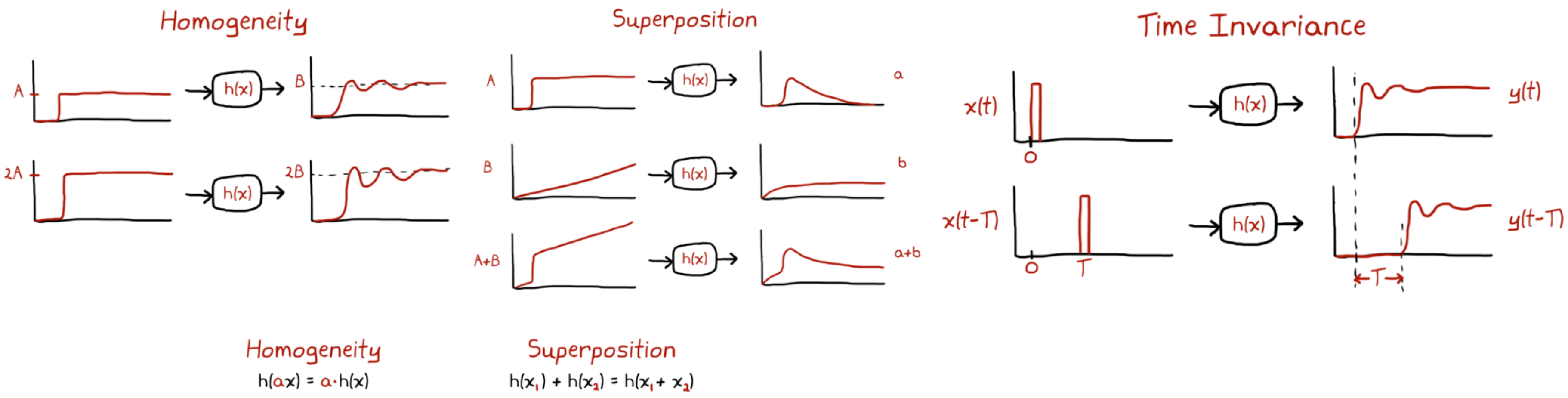
- Linearity is defined in terms of the input $u(t)$ and output $y(t)$
  - Also known as excitation and response.
- If a system satisfies **homogeneity** and **superposition**, it is a linear system.
- If a linear system further sanctifies the property of **time-invariance**, it is then called an LTI system.
  - Linearity + Time-invariance = **sinusoidal fidelity**
  - Linear system is a wider concept than *linear time-invariant (LTI)* system, also including *linear time-varying* system



2稳态响应： 理解稳态误差对应的表

Steady State Error Specifications

- **System type**: the number of integrators in the forward path
  - Three error constants are defined for each system type.

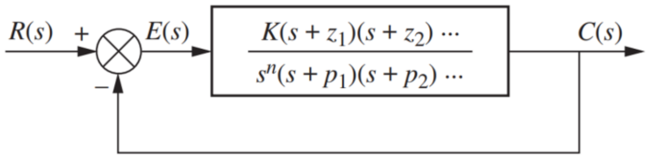


FIGURE 7.8 Feedback control system for defining system type

TABLE 7.2 Relationships between input, system type, static error constants, and steady-state errors

		Type 0 $\frac{K(s+z_1)(s+z_2)\cdots}{s^0(s+p_1)(s+p_2)\cdots}$		Type 1 $\frac{K(s+z_1)(s+z_2)\cdots}{s^1(s+p_1)(s+p_2)\cdots}$		Type 2 $\frac{K(s+z_1)(s+z_2)\cdots}{s^2(s+p_1)(s+p_2)\cdots}$	
Input	**Steady-state error formula**	**Static error constant**	**Error**	**Static error constant**	**Error**	**Static error constant**	**Error**
Step, $u(t)$	$\frac{1}{1+\lim_{s\to 0} G(s)} = \frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{\lim_{s\to 0} s^2G(s)} = \frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

Interpreting the Steady-State Error Specification

PROBLEM: What information is contained in the specification $K_p = 1000$?

SOLUTION: The system is stable. The system is Type 0, since only a Type 0 system has a finite K_p. Type 1 and Type 2 systems have $K_p = \infty$. The input test signal is a step, since K_p is specified. Finally, the error per unit step is

$$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+1000} = \frac{1}{1001} \quad (7.54)$$

3状态空间：可控性、可观性的含义，部分可控系统的极点配置

Partially Controllable System

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$$\dot{x} = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} u$$

- Further consider this state space model and do what we have done again.
- Write out the differential equation for x_3 and x_4:
  - $\dot{x}_3 = -2x_3 + 1u$
  - $\dot{x}_4 = -2x_4 + 2u$
  - Define a new state denoted by $\Delta x = 2x_3 - x_4$,
  - which is governed by $\frac{\mathrm{d}}{\mathrm{dt}}\Delta x = -4x_3 + 2x_4 = -2\Delta x$
  - The eigenvalue of the new state Δx cannot be modified by u.

    - This is again what essentially the concept of uncontrollable state.
- For the first block, let's also cancel the input u by defining a new delta state.
  - $\frac{\mathrm{d}}{\mathrm{dt}}\Delta x = \frac{\mathrm{d}}{\mathrm{dt}}(x_1 - x_2) = 3x_1 + x_2 - 3x_2 = -3\Delta x + x_2$
  - Here, x_2 is affected by u and can be treated as a virtual input.

Controllability

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A key question that arises in the design of state variable compensators is whether or not all the poles of the closed-loop system can be arbitrarily placed in the complex plane.

if the system is controllable and observable, then we can.

The concepts of controllability and observability were introduced by Kalman in the 1960s:

A system is completely controllable if there exists an unconstrained control $u(t)$ that can transfer any initial state $x(t_0)$ to any other desired location $x(t)$ in a finite time, $t_0 \le t \le T$.

For the SISO LTI system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t),$$

we can determine whether the system is controllable by examining the algebraic condition

$$\operatorname{rank}[\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \ldots \mathbf{A}^{n-1}\mathbf{B}] = n.$$

dimension of the system.

The controllability matrix P_c is described in terms of A and B as

$$\mathbf{P}_c = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \ldots \mathbf{A}^{n-1}\mathbf{B}],$$

If the determinant of P_c is nonzero, the system is controllable.

4频率响应：波特图、相位裕度与非最小相位系统

- 相位裕度的正负对于非最小相位系统来说是没意义的

Stability Margin in terms of F.R.

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- When we are discussing stability, we are implying the stability of the closed loop control system.
- The frequency response of a closed loop transfer function can be derived as follows.

$$T(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$\Rightarrow T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}$$

- Note this $T(j\omega)$ is the complex gain to a sinusoidal input $R(j\omega)$ and **apparently this gain should not be equal to infinity (as a sufficient condition),** that is, the open loop transfer function should satisfy:

$$\Rightarrow L(j\omega) \triangleq G(j\omega)H(j\omega) \neq -1 = 1\angle 180^\circ$$

Gain margin, G_M. The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180° of phase shift to make the closed-loop system unstable.

Phase margin, Φ_M. The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

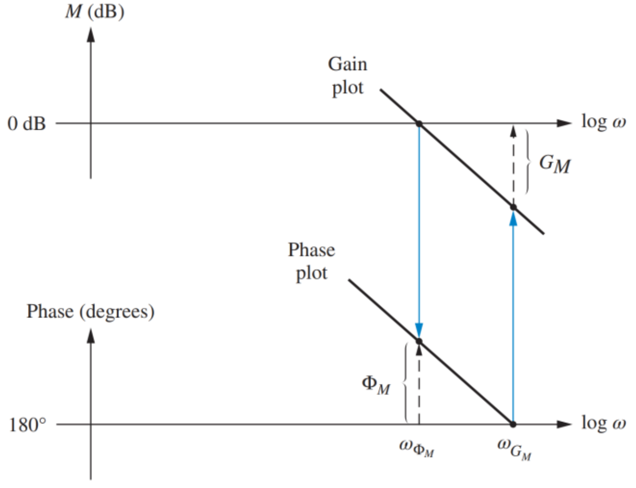

FIGURE 10.37 Gain and phase margins on the Bode diagrams

Nonminimum Phase System due to RHP Zero/Pole

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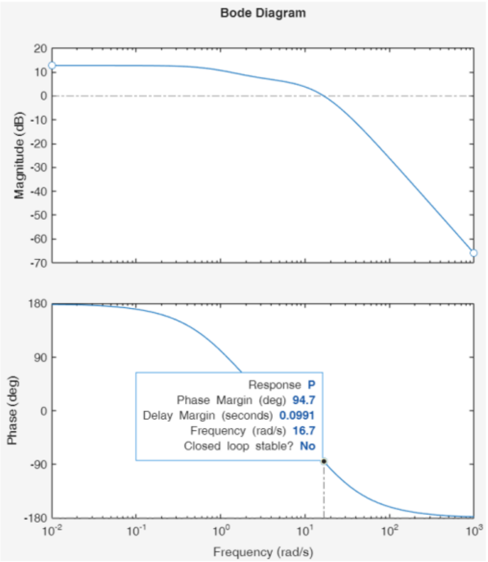
- As a motivation, consider a transfer function, and if its open loop pole or zero is mirrored about the $j\omega$-axis, i.e., it is moved to the right hand s-plane, the new transfer function shares the same **magnitude** frequency response but its **phase** frequency response would become more lagging or advancing over frequency variable ω.
- An example is provided as script below. Can you check its stability and if it is unstable can you make it stable?

```
close all; cla; clc
s = zpk(0, [], 1);
Popen = 500 * (s+2)/(s+1)/(s^2+30*s+229)
Popen = 500 * (s-2)/(s+1)/(s^2+30*s+229)
P = Popen
allmargin(Popen)
h = bodeplot(Popen);
h.showCharacteristic('AllStabilityMargins')
step(Popen)
P = Popen/(1+Popen); step(P)
```



5. 数学建模：传递函数、状态空间、框图，三者之间相互转换

Block diagram reduction

- Feedback

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

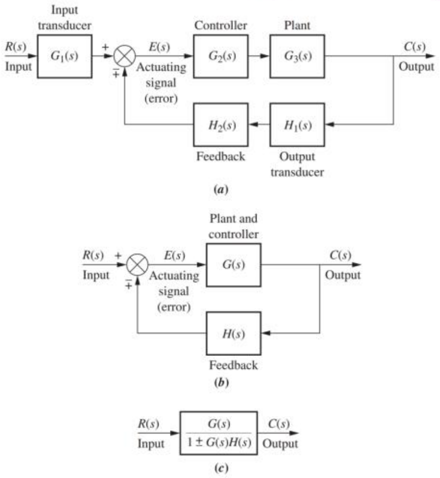


FIGURE 5.6 **a.** Feedback control system; **b.** simplified model; **c.** equivalent transfer function

Transfer Function

- General case of a high order differential equation:

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1}c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}} + \cdots + b_0 r(t) \quad (2.50)$$

- **Convert it into s-domain:**

$$a_n s^n C(s) + a_{n-1}s^{n-1}C(s) + \cdots + a_0 C(s) + \text{initial condition terms involving } c(t)$$
$$= b_m s^m R(s) + b_{m-1}s^{m-1}R(s) + \cdots + b_0 R(s) + \text{initial condition terms involving } r(t)$$

- Assume zero initial conditions we get transfer function

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0)}$$

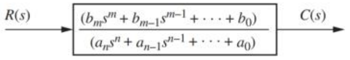


FIGURE 2.2 Block diagram of a transfer function

Norman Nise , Control system engineering (7th Edition)

From transfer function to state space model (1)

Converting a Transfer Function with a Constant Term in the Numerator

$$\frac{C(s)}{R(s)} = \frac{24}{(s^3 + 9s^2 + 26s + 24)}$$

$$(s^3 + 9s^2 + 26s + 24)C(s) = 24R(s)$$

$$\dddot{c} + 9\ddot{c} + 26\dot{c} + 24c = 24r$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 24\end{bmatrix} r$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

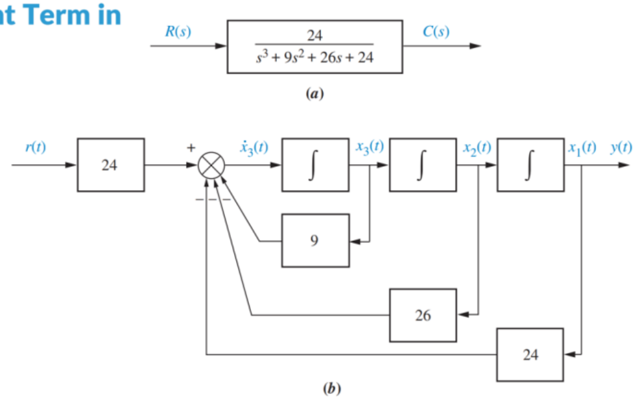


FIGURE 3.10 **a.** Transfer function; **b.** equivalent block diagram showing phase variables. Note: $y(t) = c(t)$.

6. 稳定性： 劳斯判据

Routh-Hurwitz Criterion

- Simply stated, the **Routh-Hurwitz criterion** declares that the number of roots of the polynomial that are in the right half-plane is equal to the number of sign changes in the first column of Routh table

PROBLEM: Make a Routh table and tell how many roots of the following polynomial are in the right half-plane and in the left half-plane.

$$P(s) = 3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6$$

ANSWER:

Routh-Hurwitz Table Generator

Polynomial

Enter polynomial coefficients separated by commas

6,2,8,7,4,6,9,3

$$3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6$$

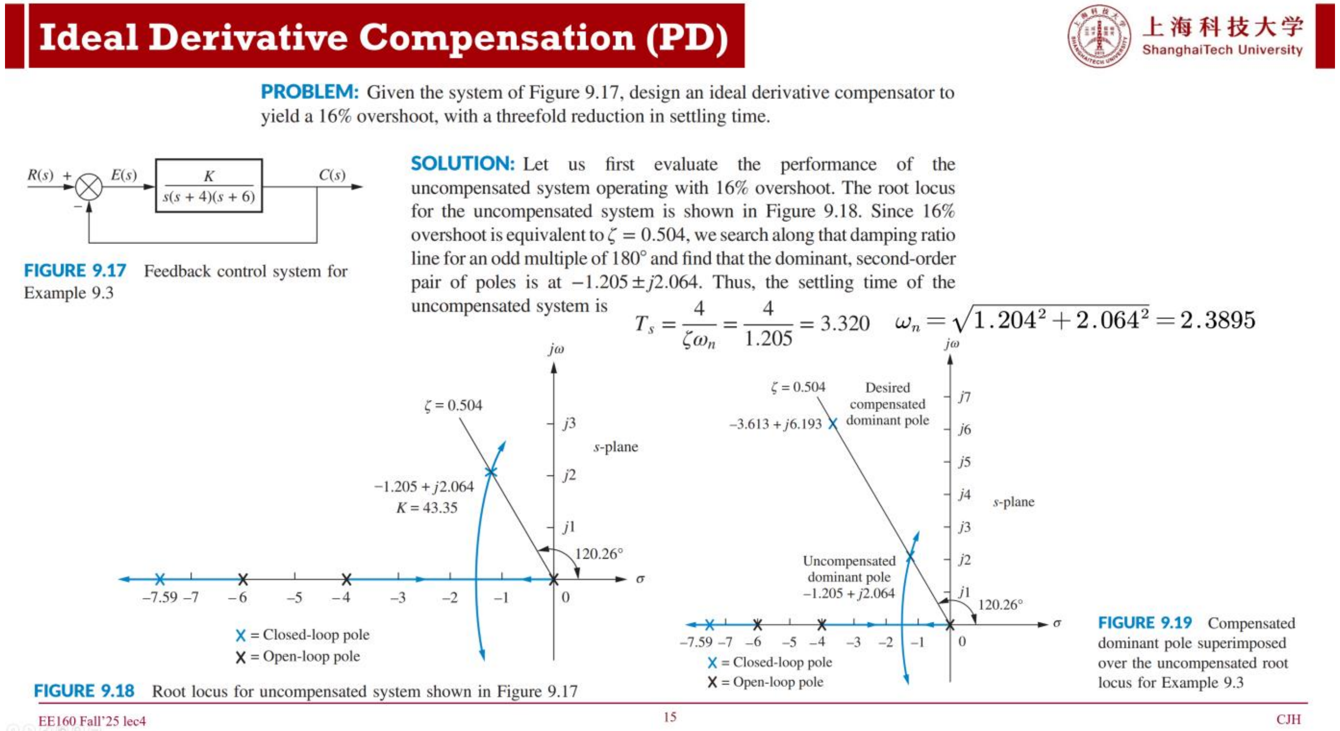
Go

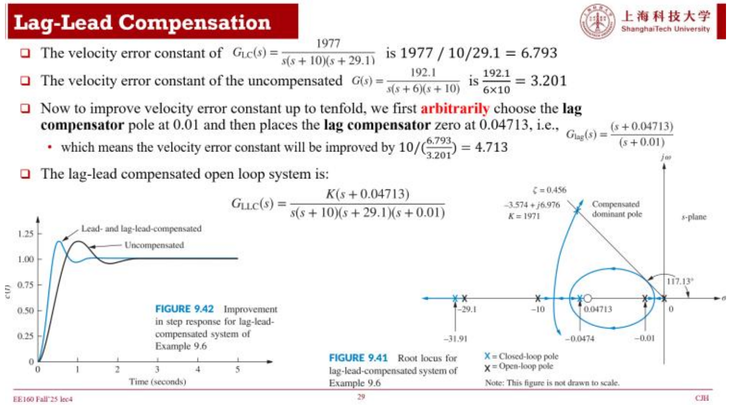
Routh-Hurwitz Table

s^7	3	6	7	2
s^6	9	4	8	6
s^5	$\frac{14}{3}$	$\frac{13}{3}$	0	0
s^4	$-\frac{61}{14}$	8	6	0
s^3	$\frac{787}{61}$	$\frac{392}{61}$	0	0
s^2	$\frac{8004}{787}$	6	0	0
s	$-\frac{1581}{1334}$	0	0	0
1	6	0	0	0

https://routhhurwitz.streamlit.app/

7. 瞬态响应：校正器设计（根轨迹法





猜测ω_n，也就是主导极点距离原点的距离，

未动态校正的根这样算：$s=\omega_n\left(\zeta+j\sqrt{1-\zeta^2}\right)$

特征方程：$1+G(s)=0$

$$\Rightarrow G(s)=\frac{K}{s(s+4)(s+6)}=-1$$

验证s是否符合角度条件：

$$\Rightarrow \angle G(s)=-\angle s(s+4)(s+6)=180^\circ$$

确定s的值，然后计算希望动态校正到哪里：

$\square$ = 横坐标 + j纵坐标

- 横坐标基于调节时间T_s给定，然后根据ζ斜线求纵坐标
- 纵坐标基于峰值时间T_p给定，然后根据ζ斜线求恒坐标

特征方程：$1+L(s)=0$

$$\Rightarrow L(s)=C(s)G(s)=\frac{(s+z_c)}{(s+p_c)}\frac{K}{s(s+4)(s+6)}=-1$$

根据角度条件求PD控制器的零点放哪里：

$$\Rightarrow \angle L(\square)=\angle(\square+z_c)-(\square+p_c)-\angle\square(\square+4)(\square+6)=180$$

$$\Rightarrow \angle(\square+z_c)-(\square+p_c)=180^\circ+\angle\square(\square+4)(\square+6)$$

和频域波特图法）

My Personally Recommended Standard Procedure

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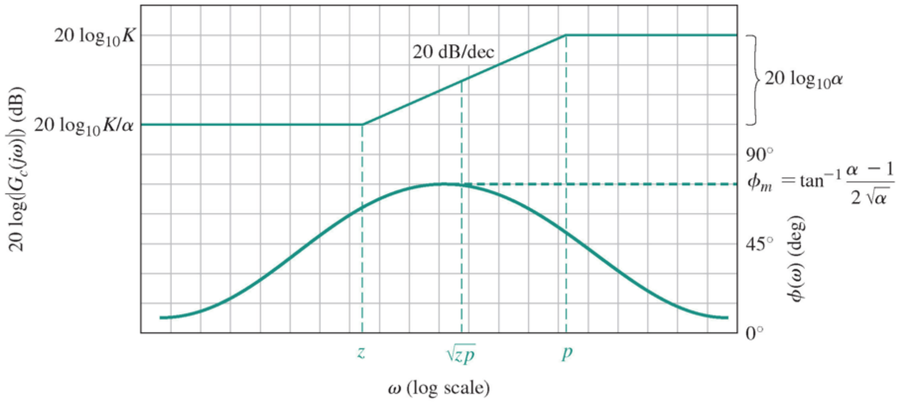
- Consider a unity negative feedback system with a plant $P(s) = \frac{1}{s}\frac{1}{0.1s+1}\frac{1}{0.05s+1}$.
  - Try to design cascade compensator such that the **velocity constant** is larger than 20, the **corner frequency** is at least 30 rad/s, and the **phase margin** is at least 50°.
  - Hint: if you find one lead compensator is not enough, consider adding one more.
- The lag compensator for steady state error is designed after the phase margin requirement has been met.




```
close all;clc;clear all;
s = tf('s');
P = 1 / (s) / (0.05*s+1) / (0.1*s+1);
wc = 30;
K = 10^(-10/20) / abs(evalfr(P, wc*1j))
figure();
bode(K*P)
wm = wc;
alpha = 10;
z1 = sqrt(wm^2/alpha);
p1 = alpha * z1;
C = (s/z1+1) / (s/p1+1);
figure();
bode(C)
OL = C*K*P;
figure();
bode(OL)
allmargin(OL)
```

The maximum value of the phase lead occurs at:

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$