上海科技大学
ShanghaiTech University

Lecture 6: Gang of Six

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SIST 1D#206

Control System with Multiple Inputs

- The gang of six

$$X = \frac{CP}{1+CP}FR + \frac{P}{1+CP}D - \frac{CP}{1+CP}N$$

$$Y = \frac{CP}{1+CP}FR + \frac{P}{1+CP}D + \frac{1}{1+CP}N$$

$$U = \frac{C}{1+CP}FR - \frac{CP}{1+CP}D - \frac{C}{1+CP}N$$

$$E = \frac{1}{1+CP}FR - \frac{1P}{1+CP}D - \frac{1}{1+CP}N$$

$$\begin{aligned} U = CE &= C\left(FR - Y\right) \\ &= C\left(FR - \frac{CP}{1+CP}FR - \frac{P}{1+CP}D - \frac{1}{1+CP}N\right) \\ &= \left(\frac{C}{1+CP}FR - \frac{CP}{1+CP}D - \frac{C}{1+CP}N\right) \end{aligned}$$

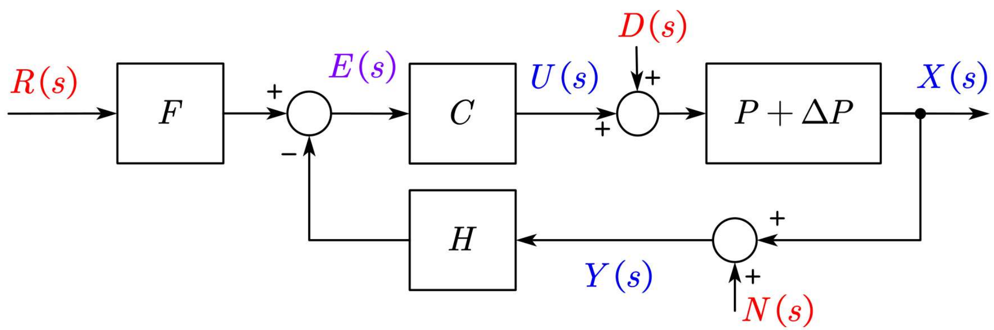


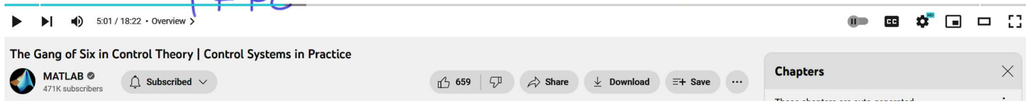
Figure 17. Closed loop system with three input channels.

$\frac{C}{1+PC}$ Noise sensitivity function

$\frac{P}{1+PC}$ Disturbance sensitivity function

$\frac{1}{1+PC}$ Sensitivity function

$\frac{PC}{1+PC}$ Complementary sensitivity function



Sensitivity Function

Sensitivity Function

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- Sensitivity is the **ratio of** the fractional change in the function **to** the fractional change in the parameter **as** the fractional change of the parameter approaches zero.

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\text{Fractional change in the function, } F}{\text{Fractional change in the parameter, } P}$$

$$= \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P}$$

$$= \lim_{\Delta P \to 0} \frac{P \Delta F}{F \Delta P}$$

$$S_{F:P} = \frac{P}{F}\frac{\delta F}{\delta P}$$

- Alternatively, parameter sensitivity can be calculated as follows:

$$T(s;\alpha) = \frac{N(s;\alpha)}{D(s;\alpha)}$$

$$S_\alpha^T = \frac{\partial \ln \mathrm{T}}{\partial \ln \alpha} = \left.\frac{\partial \ln \mathrm{N}}{\partial \ln \alpha}\right|_{\alpha=\alpha_0} - \left.\frac{\partial \ln \mathrm{D}}{\partial \ln \alpha}\right|_{\alpha=\alpha_0}$$

Sensitivity of a Closed-Loop Transfer Function

PROBLEM: Given the system of Figure 7.19, calculate the sensitivity of the closed-loop transfer function to changes in the parameter a. How would you reduce the sensitivity?

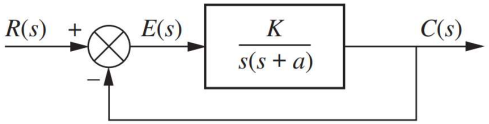

FIGURE 7.19 Feedback control system for Examples 7.10 and 7.11

SOLUTION: The closed-loop transfer function is

$$T(s) = \frac{K}{s^2 + as + K} \tag{7.76}$$

Using Eq. (7.75), the sensitivity is given by

$$S_{T:a} = \frac{a}{T}\frac{\delta T}{\delta a} = \frac{a}{\left(\frac{K}{s^2+as+K}\right)}\left(\frac{-Ks}{(s^2+as+K)^2}\right) = \frac{-as}{s^2+as+K} \tag{7.77}$$

which is, in part, a function of the value of s. For any value of s, however, an increase in K reduces the sensitivity of the closed-loop transfer function to changes in the parameter a.

Sensitivity Function of a Closed Loop Control System

- The sensitivity function of a closed loop system $T(s)$ with respect to the plant transfer function $P(s)$ can be derived as follows:

$$T(s) = \frac{CP}{1+CP}$$

$$\Rightarrow S_P^T = \frac{P}{T}\frac{\partial T}{\partial P} = \frac{P}{\frac{CP}{1+CP}} \frac{\partial \frac{CP}{1+CP}}{\partial P} = \frac{1+CP}{C} \frac{C(1+CP) - CP(C)}{(1+CP)^2} = \frac{1}{1+CP}$$

  - S happens to be the transfer function from noise to output.
  - S is the fundamental reason why negative feedback system is superior than an open loop control system, in a sense that when the plant parameters deviate over time, the output is minimally affected.

Error Signal Analysis

(Reference tracking, disturbance rejection, and noise attenuation)

(Review) Steady State Error due to Disturbance

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- Review the problem formulation of a control system two input channels: the reference signal $R(s)$ and disturbance signal $D(s)$

$$C(s) = E(s)G_1(s)G_2(s) + D(s)G_2(s)$$

$$C(s) = R(s) - E(s)$$

$$E(s) = \frac{1}{1 + G_1(s)G_2(s)}R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)}D(s)$$

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s}{1 + G_1(s)G_2(s)}R(s) - \lim_{s\to 0}\frac{sG_2(s)}{1 + G_1(s)G_2(s)}D(s) = e_R(\infty) + e_D(\infty)$$

- For a step input of $D(s) = \frac{1}{s}$ the steady state error is

$$e_D(\infty) = -\frac{1}{\lim_{s\to 0}\frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)}$$

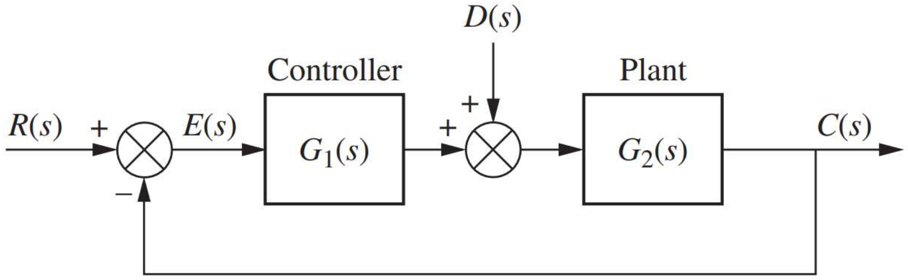

FIGURE 7.11 Feedback control system showing disturbance

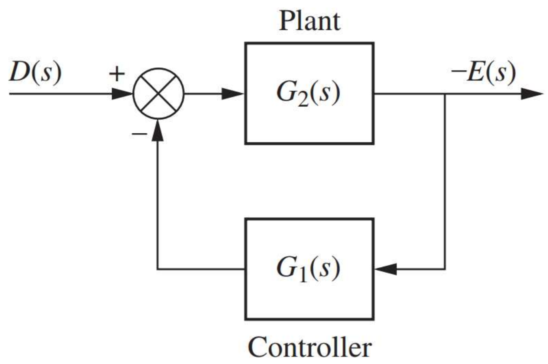

FIGURE 7.12 Figure 7.11 system rearranged to show disturbance as input and error as output, with $R(s) = 0$

Eliminating Steady State Error due to Disturbance

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- For the problem shown in the right, can you design a cascaded compensator that eliminate the steady state error due to disturbance, $e_D(\infty)$?
  - The PI regulator can add an integrator in the feedback path of the closed loop system with disturbance as input and error as output, such that $G_1(0) = \infty$

$$e_D(\infty) = -\frac{1}{\lim_{s\to 0}\frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)}$$

PROBLEM: Find the steady-state error component due to a step disturbance for the system of Figure 7.13.

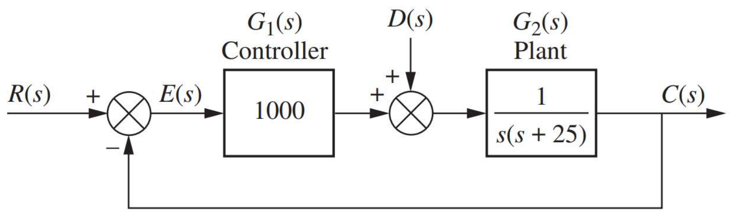


FIGURE 7.13 Feedback control system for Example 7.7

SOLUTION: The system is stable. Using Figure 7.12 and Eq. (7.62), we find

$$e_D(\infty) = -\frac{1}{\lim_{s\to 0}\frac{1}{G_2(s)} + \lim_{s\to 0} G_1(s)} = -\frac{1}{0 + 1000} = -\frac{1}{1000} \tag{7.63}$$

The result shows that the steady-state error produced by the step disturbance is inversely proportional to the dc gain of $G_1(s)$. The dc gain of $G_2(s)$ is infinite in this example.

Frequency Response of typical C.L. tf's

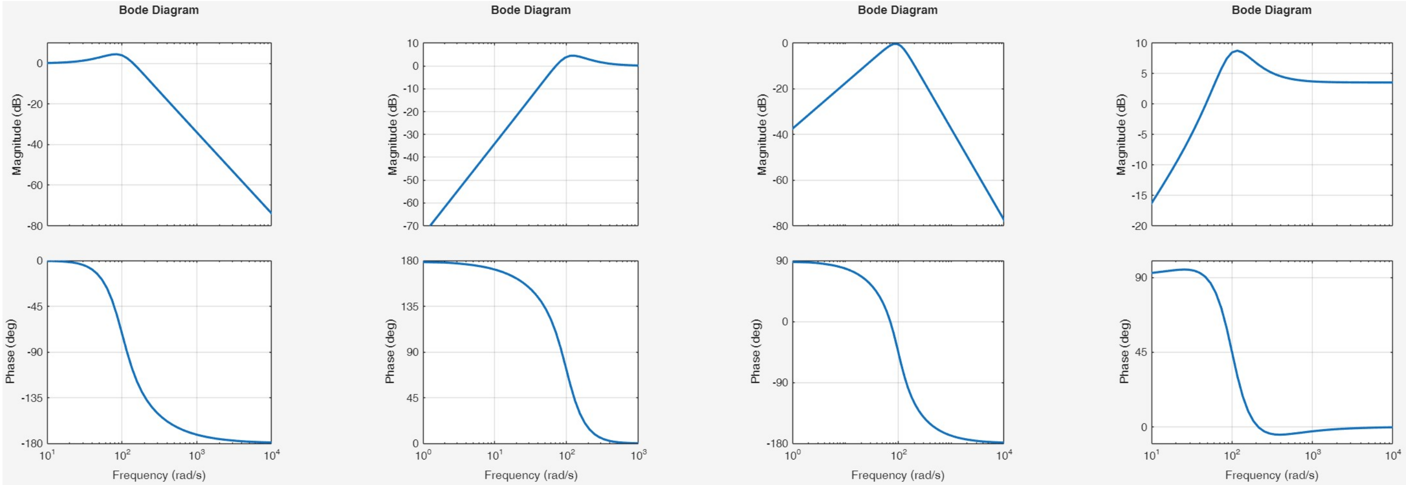
- Revisit the serve control system. Four of the transfer functions are evaluated for frequency response:

$$X = \frac{CP}{1+CP}FR + \frac{P}{1+CP}D - \frac{CP}{1+CP}N$$

$$Y = \boxed{\frac{CP}{1+CP}FR} + \boxed{\frac{P}{1+CP}D} + \boxed{\frac{1}{1+CP}N}$$

$$U = \frac{C}{1+CP}FR - \frac{CP}{1+CP}D - \frac{C}{1+CP}N$$

$$E = \boxed{\frac{1}{1+CP}FR} - \frac{1P}{1+CP}D - \boxed{\frac{1}{1+CP}N}$$




```
figure(2)
C = CVL;
P = PclosedInner * PVL;

subplot(141); bode(C*P/(1+C*P)); grid; h1 = findobj(gcf,'type','line'); set(h1,'linewidth',2);
subplot(142); bode(1/(1+C*P)); grid; h1 = findobj(gcf,'type','line'); set(h1,'linewidth',2);
subplot(143); bode(P/(1+C*P)); grid; h1 = findobj(gcf,'type','line'); set(h1,'linewidth',2);
subplot(144); bode(C/(1+C*P)); grid; h1 = findobj(gcf,'type','line'); set(h1,'linewidth',2);
```

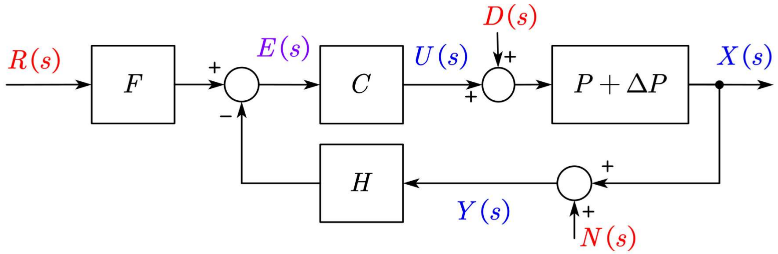


Figure 17. Closed loop system with three input channels.

Frequency Response of typical C.L. tf's

- Example paper using frequency responses to evaluate a servo control system

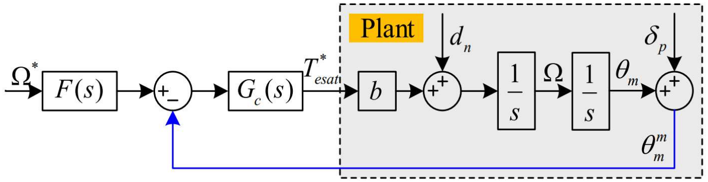


Fig. 3. Block diagram of different ADRC systems in transfer function form

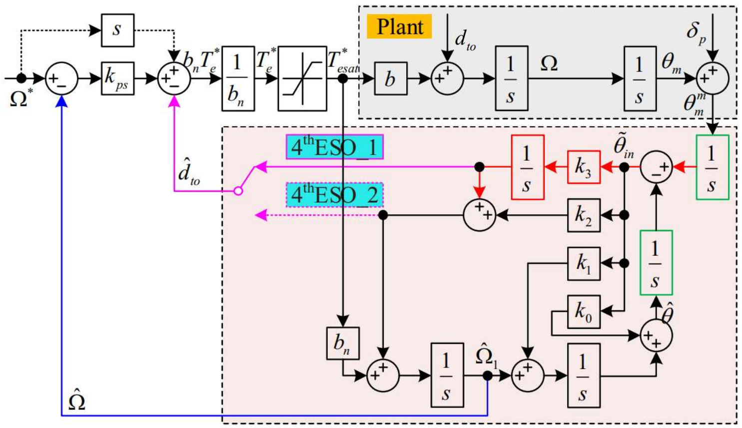


Fig. 14. The equivalent block diagram of the system 4^{th}ESO_1-2.

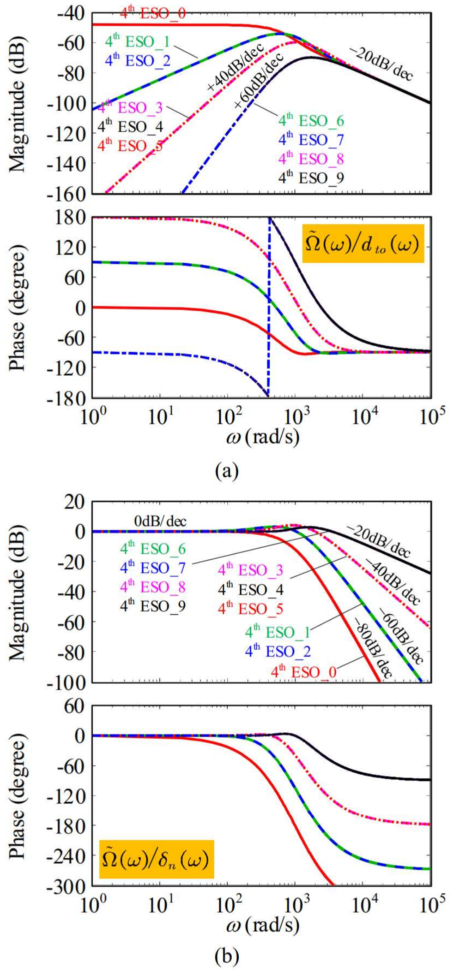


Fig. 10. Bode diagrams of the ten fourth order ESOs for observing speed.

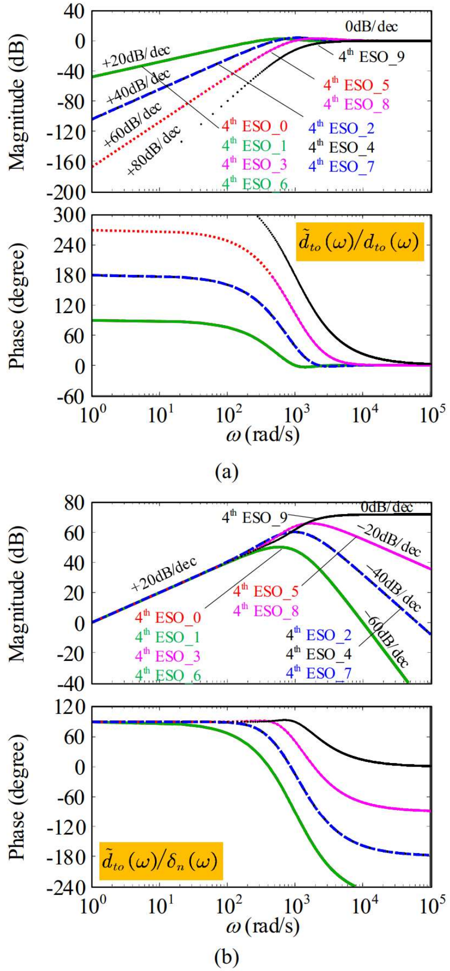


Fig. 11. Bode diagrams of the ten fourth order ESOs for observing disturbance.

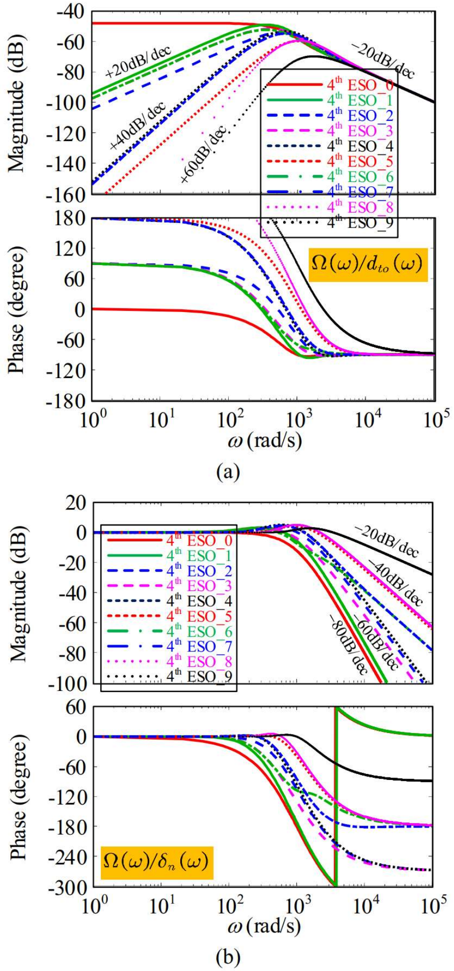


Fig. 12. Bode diagram of the ten systems under the same p_o.

Zuo et al., "Different Active Disturbance Rejection Controllers Based on the Same Order GPI Observer", TIE, 2021

Two Degree-of-freedom Control

- Two degree of freedom PI control is able to eliminate overshoot by equivalently making the reference tracking transfer function a first order system.

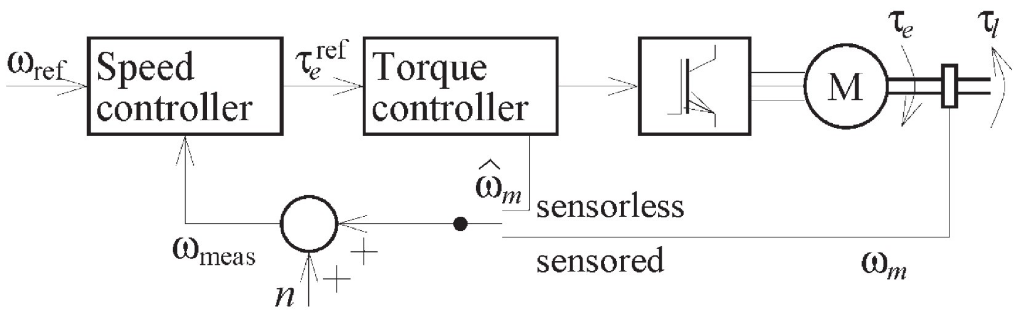


Fig. 1. Illustration of the speed control loop. The torque controller computes switching commands to the converter, which, in turn, powers the (M) motor.

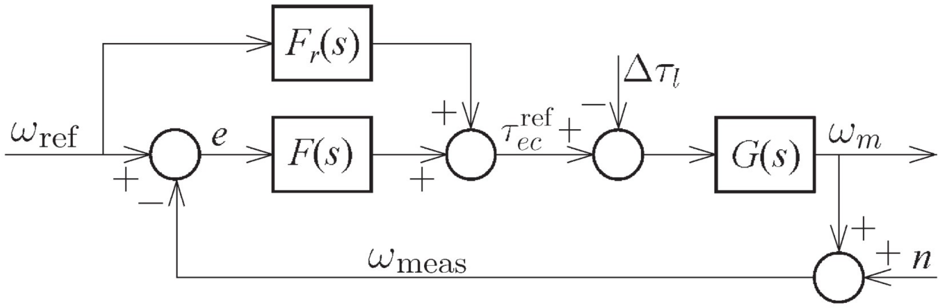


Fig. 4. Block diagram of the 2DOF structure.

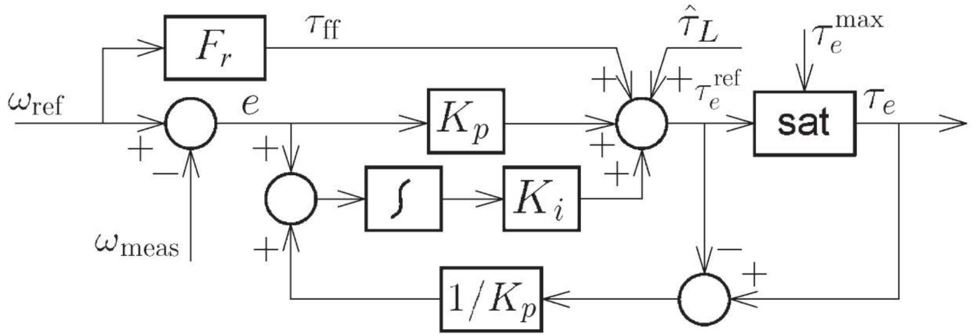


Fig. 8. Block diagram of the proposed 2DOF PI controller.

HARNEFORS *et al.*: SPEED CONTROL OF ELECTRICAL DRIVES USING CLASSICAL CONTROL METHODS, TIE, 2013

The transfer function from load torque disturbance $-T_L(s)$ to speed output $\Omega(s)$ can be analyzed using the following code snippet.

```
openloopDisturbance = 1/s / Js;
H = CVL * PclosedInner;
PDisturbance = openloopDisturbance/(1+openloopDisturbance*H)
figure; bode(PDisturbance); grid; h1 = findobj(gcf,'type','line');
    set(h1,'linewidth',2);
```

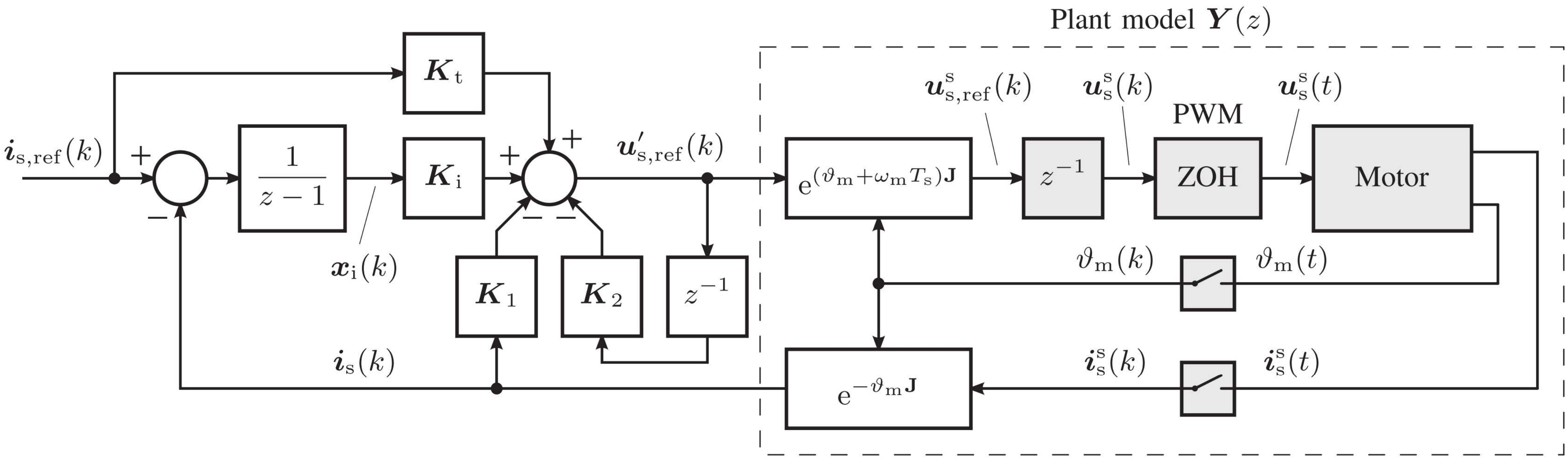


Fig. 1. State-feedback current controller with integral action and reference feedforward. The gray blocks represent the physical system (including the motor, PWM, samplers, and inherent computational delay z^{-1}). The block "Motor" consists of (2), (3), and the coordinate transformations. The PWM is modeled as the ZOH in stator coordinates. The sampling of the stator currents is synchronized with the PWM. The white blocks represent the discrete-time control algorithm. The angular error due to the time delay is compensated for in the coordinate transformation of the stator voltage.