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Lecture 5: Frequency Response Based Compensator Design

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SIST 1D#206

Frequency Response
(The study of system in terms of input and output)

Frequency Response

- Consider a sinusoidal input to a transfer function G(s), and its output is:

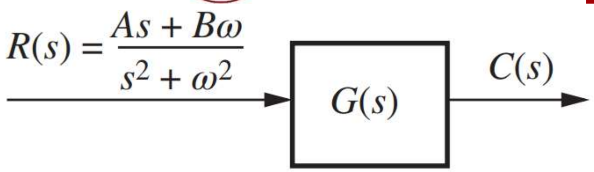

FIGURE 10.3 System with sinusoidal input

$$C(s) = \frac{As + B\omega}{(s^2 + \omega^2)} G(s) \qquad M_i = \sqrt{A^2 + B^2} \qquad \phi_i = -\tan^{-1}(B/A)$$

$$C(s) = \frac{As + B\omega}{(s + j\omega)(s - j\omega)} G(s)$$

$$= \frac{K_1}{s + j\omega} + \frac{K_2}{s - j\omega} + \text{Partial fraction terms from } G(s)$$

Steady state

$$C_{ss}(s) = \frac{K_1}{s + j\omega} + \frac{K_2}{s - j\omega}$$

where

$$K_1 = \frac{As + B\omega}{s - j\omega} G(s)\Big|_{s \to -j\omega} = \frac{1}{2}(A + jB)G(-j\omega) = \frac{1}{2} M_i e^{-j\phi_i} M_G e^{-j\phi_G}$$

$$= \frac{M_i M_G}{2} e^{-j(\phi_i + \phi_G)}$$

$$K_2 = \frac{As + B\omega}{s + j\omega} G(s)\Big|_{s \to +j\omega} = \frac{1}{2}(A - jB)G(j\omega) = \frac{1}{2} M_i e^{j\phi_i} M_G e^{j\phi_G}$$

$$= \frac{M_i M_G}{2} e^{j(\phi_i + \phi_G)} = K_1^*$$

$$C_{ss}(s) = \frac{\frac{M_i M_G}{2} e^{-j(\phi_i + \phi_G)}}{s + j\omega} + \frac{\frac{M_i M_G}{2} e^{j(\phi_i + \phi_G)}}{s - j\omega}$$

$$c(t) = M_i M_G \left(\frac{e^{-j(\omega t + \phi_i + \phi_G)} + e^{j(\omega t + \phi_i + \phi_G)}}{2} \right)$$

$$= M_i M_G \cos(\omega t + \phi_i + \phi_G)$$

Define Frequency Response

$$M_G \angle \phi_G = G(j\omega)$$

Frequency Response

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$$M_o(\omega)\angle\phi_o(\omega) = M_i(\omega)M(\omega)\angle[\phi_i(\omega) + \phi(\omega)]$$

- The frequency response is an idea of modeling the system as an gain in amplitude and a shift in phase of the input sinusoid of angular frequency of ω

$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)}$$

$$\phi(\omega) = \phi_o(\omega) - \phi_i(\omega)$$

We call $M(\omega)$ the ***magnitude*** *frequency response* and $\phi(\omega)$ the ***phase*** *frequency response*.

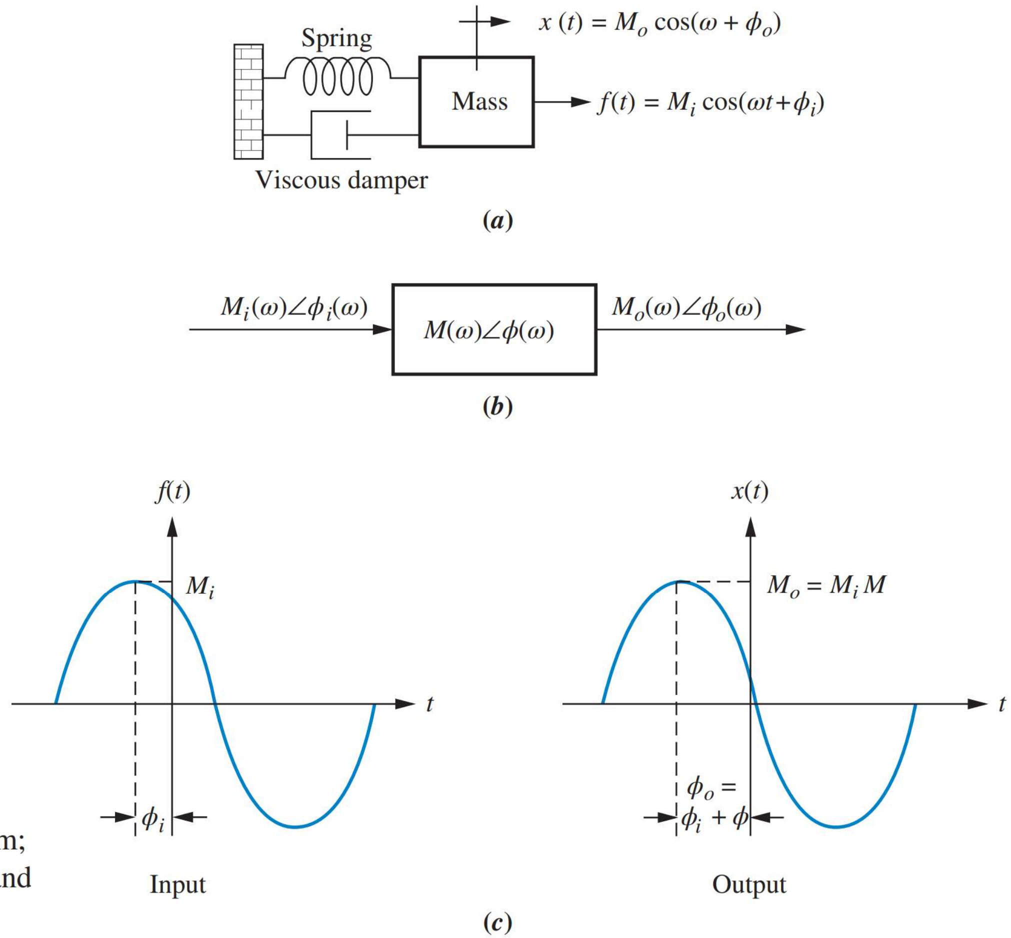


FIGURE 10.2 Sinusoidal frequency response: **a.** system; **b.** transfer function; **c.** input and output waveforms

Plotting Frequency Response

- There are two methods.
- Bode plot is
  - magnitude M in decibels (dB $= 20 \log M$) vs. $\log\omega$

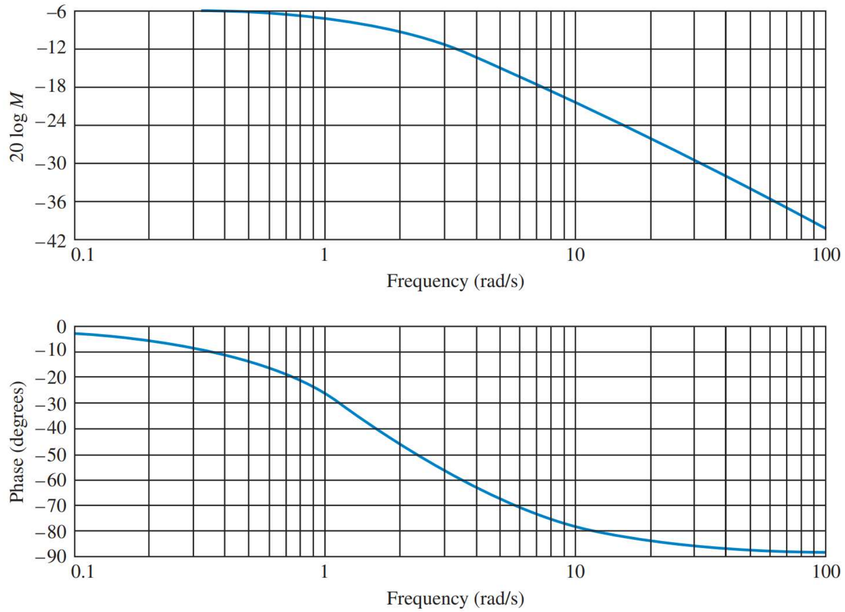
$$M(\omega) = \frac{M_o(\omega)}{M_i(\omega)}$$

  - and phase angle ϕ vs. $\log\omega$,

$$\phi(\omega) = \phi_o(\omega) - \phi_i(\omega)$$

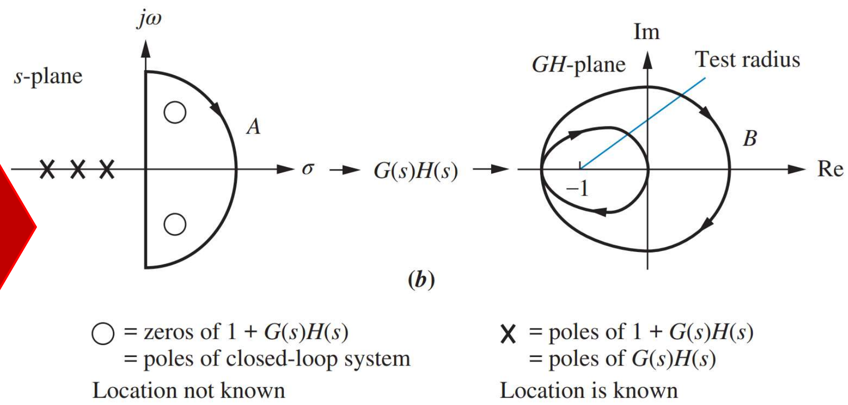
  with frequency variable ω belongs to $(0, \infty)$

Frequency variable is explicit and $\omega \in (0, \infty)$



- Nyquist plot is a polar plot, where the phasor length is the magnitude M and the phasor angle is the phase ϕ.
  - The frequency variable ω changes from $-\infty$ to $+\infty$, i.e., $\omega \in [-\infty, +\infty]$

Frequency variable is implicit and $\omega \in [-\infty, +\infty]$



Plotting Frequency Response

PROBLEM: Find the analytical expression for the magnitude frequency response and the phase frequency response for a system $G(s) = 1/(s+2)$. Also, plot both the separate magnitude and phase diagrams and the polar plot.

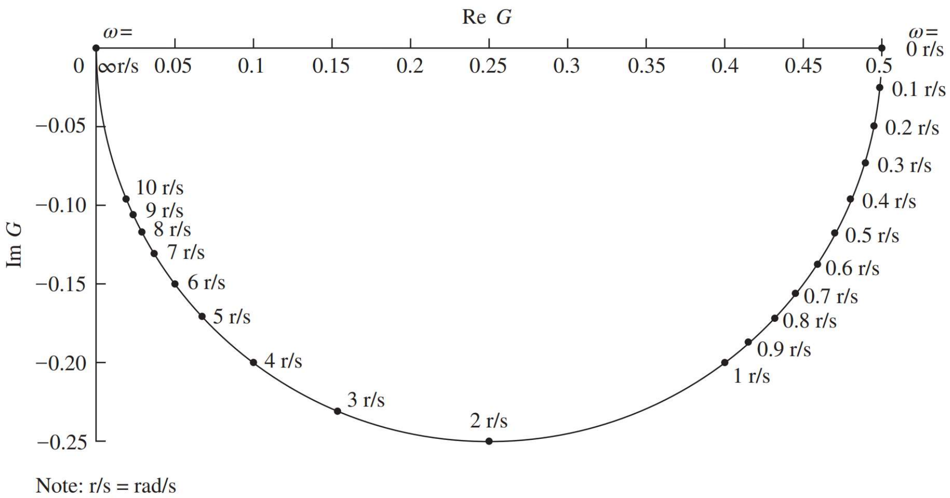


FIGURE 10.5 Frequency response plot for $G(s) = 1/(s+2)$: polar plot

$$|G(j\omega)| = M(\omega) = 1/\sqrt{(\omega^2+4)}$$

$$\phi(\omega) = -\tan^{-1}(\omega/2).$$

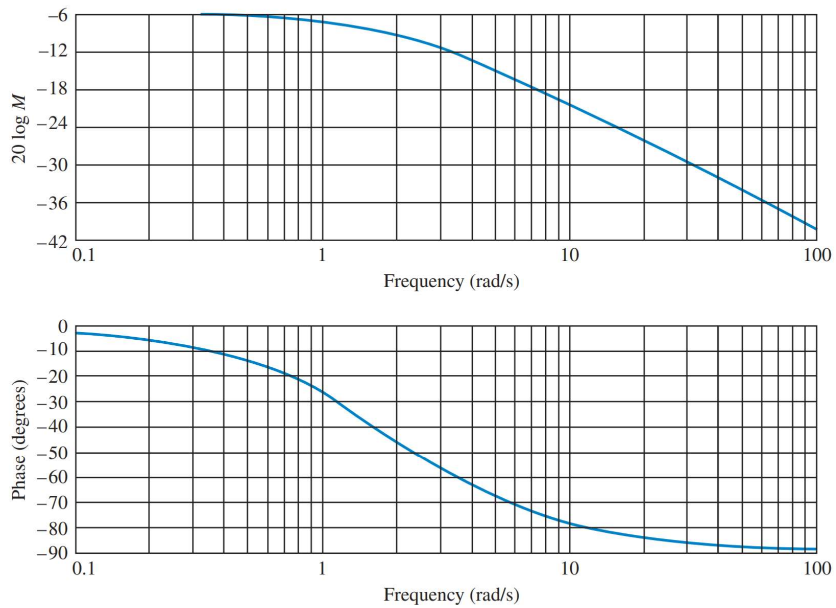


FIGURE 10.4 Frequency response plots for $G(s) = 1/(s+2)$: separate magnitude and phase diagrams

Plotting Frequency Response

PROBLEM:

a. Find analytical expressions for the magnitude and phase responses of

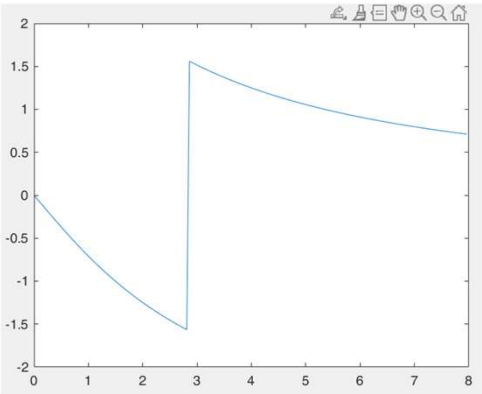
$$G(s) = \frac{1}{(s+2)(s+4)}$$

b. Make plots of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate.

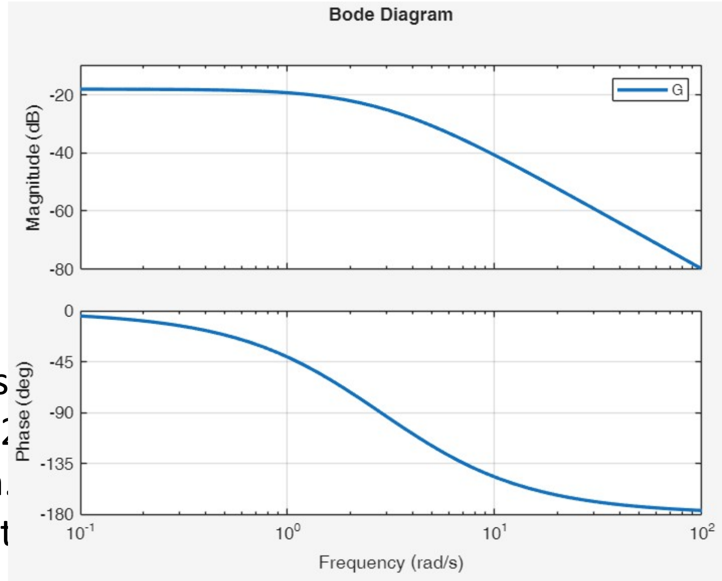
c. Make a polar plot of the frequency response.

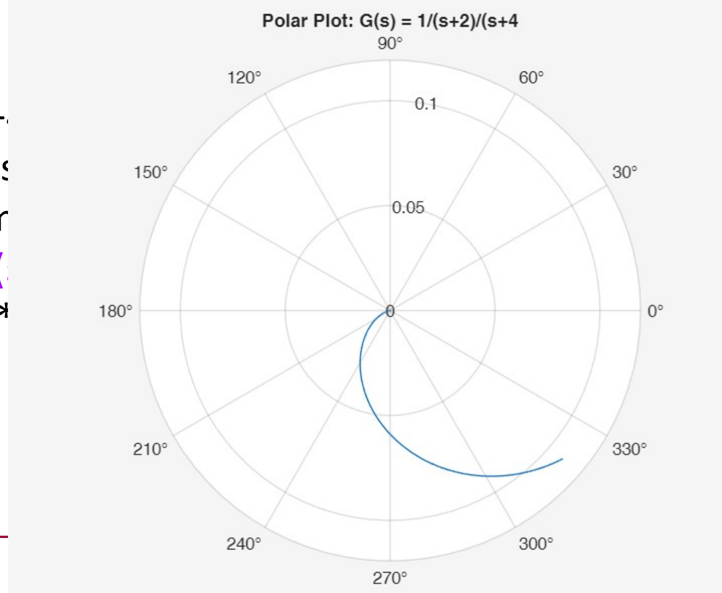
ANSWERS:

a. $M(\omega) = \dfrac{1}{\sqrt{(8-\omega^2)^2 + (6\omega)^2}}$; for $\omega \le \sqrt{8}$: $\phi(\omega) = -\arctan\left(\dfrac{6\omega}{8-\omega^2}\right)$, for $\omega > \sqrt{8}$: $\phi(\omega) = -\left[\pi + \arctan\left(\dfrac{6\omega}{8-\omega^2}\right)\right]$




```
clear;close all;clc;
s = tf('s'); G = 1 / (s
figure(1); subplot(2
h = bodeplot(G); h.
h.LegendVisible = t
grid on;
subplot(212)
w = logspace(0, 2,
s = 1j * w;
G = 1 ./ (s+2) ./ (s+
mag = abs(G); phas
polarplot(phase, m
title('Polar Plot: H(
rlim([0, max(mag)*
```





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Asymptotic Approximations of Bode Plot

Bode Plot for G(s)=(s+a)

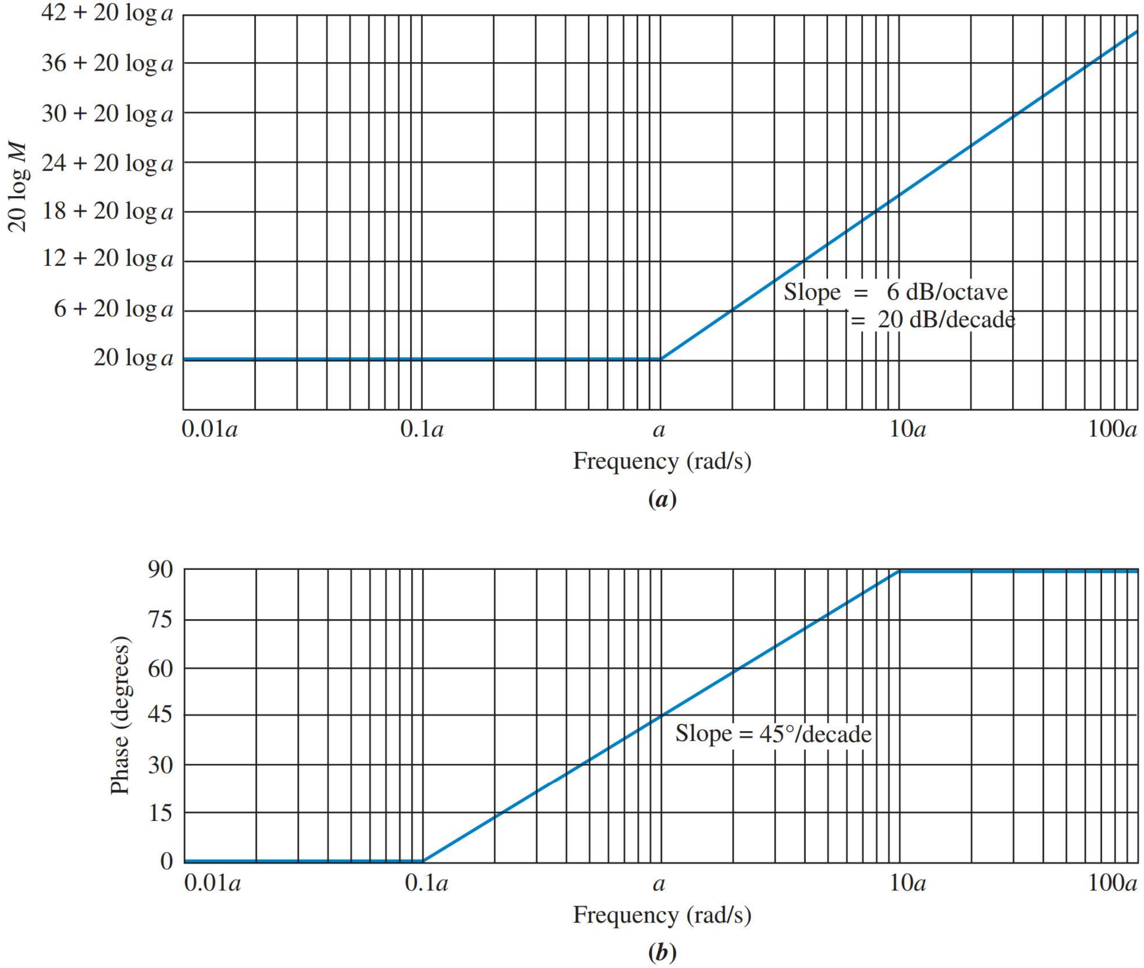

FIGURE 10.6 Bode plots of $(s + a)$: **a.** magnitude plot; **b.** phase plot

$$G(j\omega) = (j\omega + a) = a\left(j\frac{\omega}{a} + 1\right)$$

- We call the straight-line approximations *asymptotes*.
  - The low-frequency approximation is called the *low-frequency asymptote*,
  - and the high-frequency approximation is called the *high-frequency asymptote*.
- The frequency, a, is called the *break frequency* because it is the break between the low- and the high-frequency asymptotes.
- To normalize $s + a$, we factor out the quantity a and form $a(\frac{s}{a} + 1)$
  - The frequency is scaled by defining a new frequency variable, $s_1 = s/a$.
  - Then the magnitude is divided by the quantity a to yield 0 dB at the break frequency.

Bode Plot for G(s)=(s+a)

- Approximation error.
  - The actual magnitude curve is never greater than 3.01 dB from the asymptotes.
    - This maximum difference occurs at the break frequency.
  - The maximum difference for the phase curve is 5.71°,
    - which occurs at the decades above and below the break frequency.

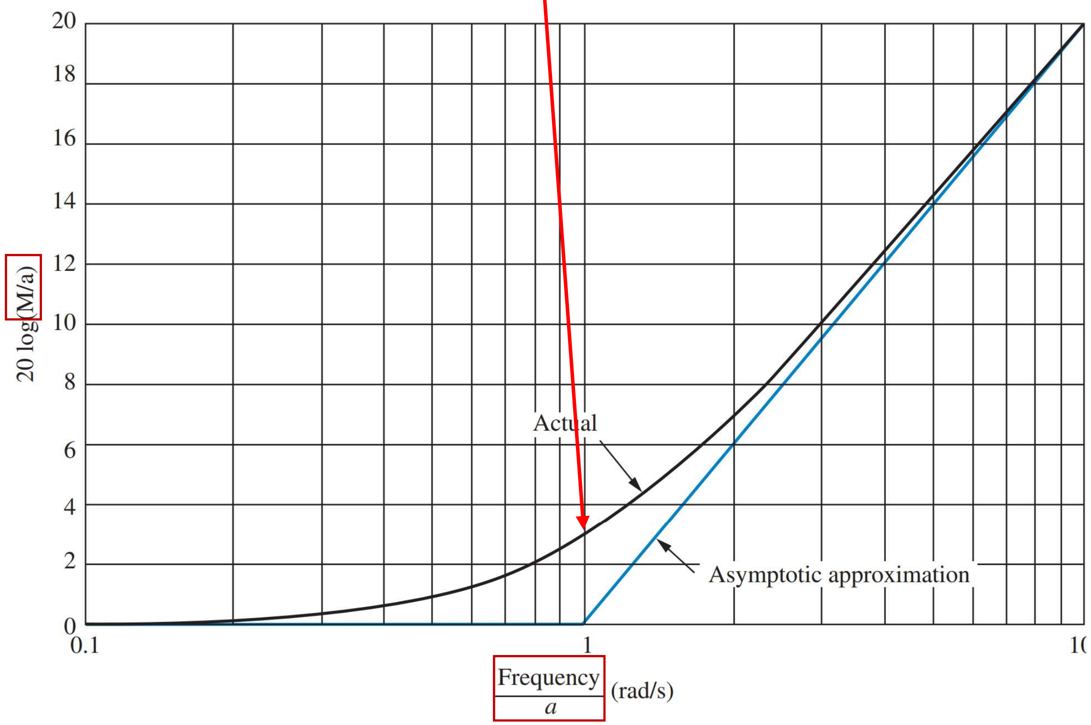

FIGURE 10.7 Asymptotic and actual normalized and scaled magnitude response of $(s + a)$

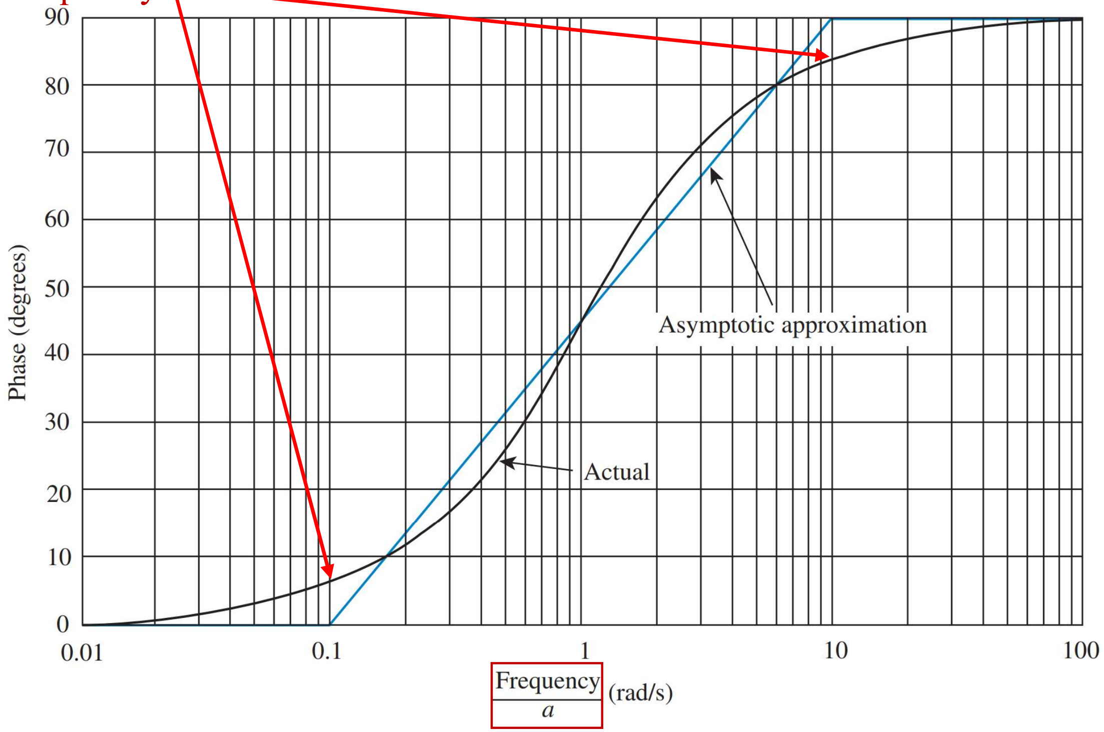

FIGURE 10.8 Asymptotic and actual normalized and scaled phase response of $(s + a)$

Typical Bode Plot Components

Bode Plot for single zero and pole

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- A **zero** or a **pole** provides:
  - a change in slope of ± 20 dB/decade at break frequency,
  - a change of $\pm 90^{\circ}$ starting at decade below break frequency and ending at a decade above the break frequency.

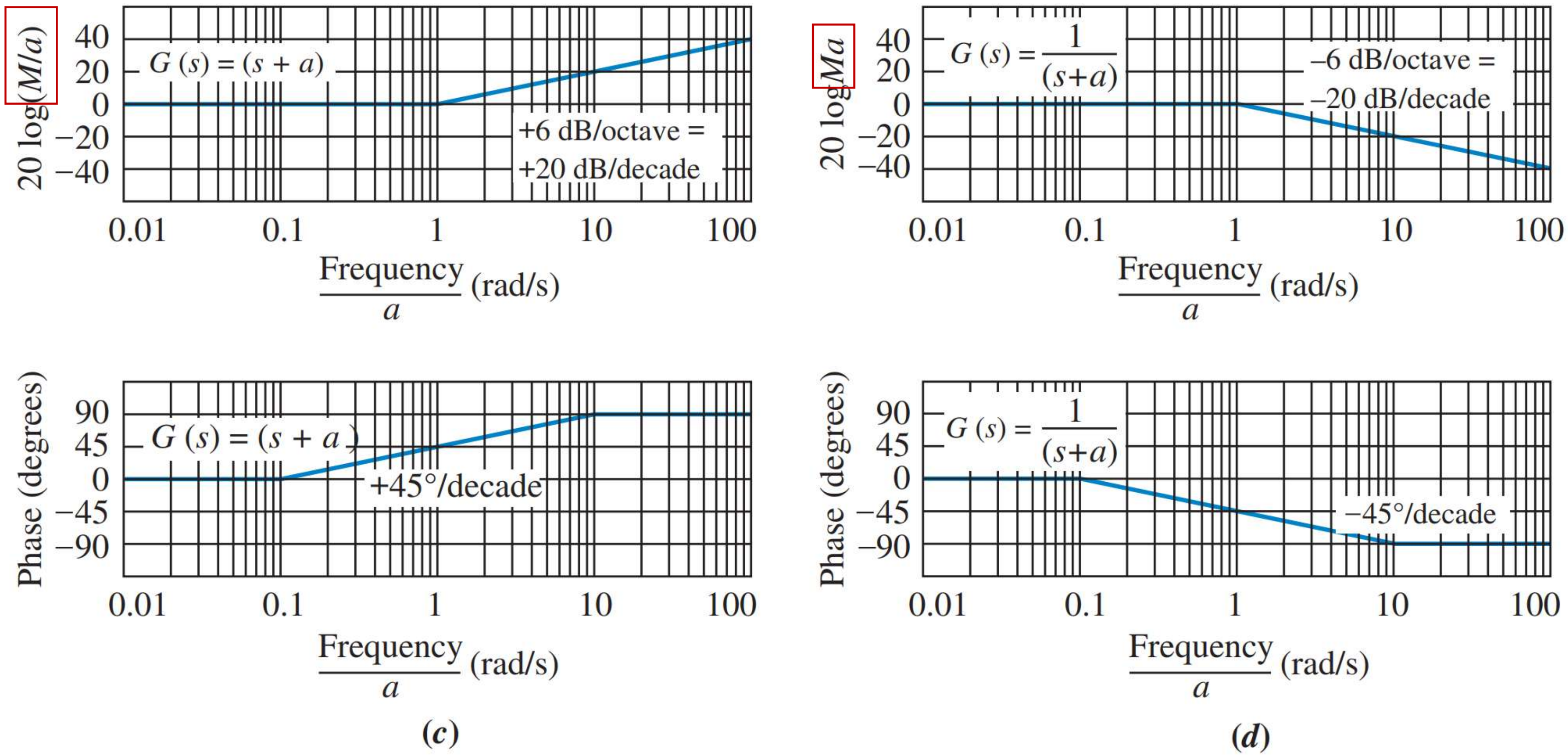

FIGURE 10.9 *(Continued)*
c. $G(s) = (s + a)$;
d. $G(s) = 1/(s + a)$

Bode Plot for single zero and pole

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- **Integrator** and **differentiator** have no break frequency and has only high-frequency asymptote:

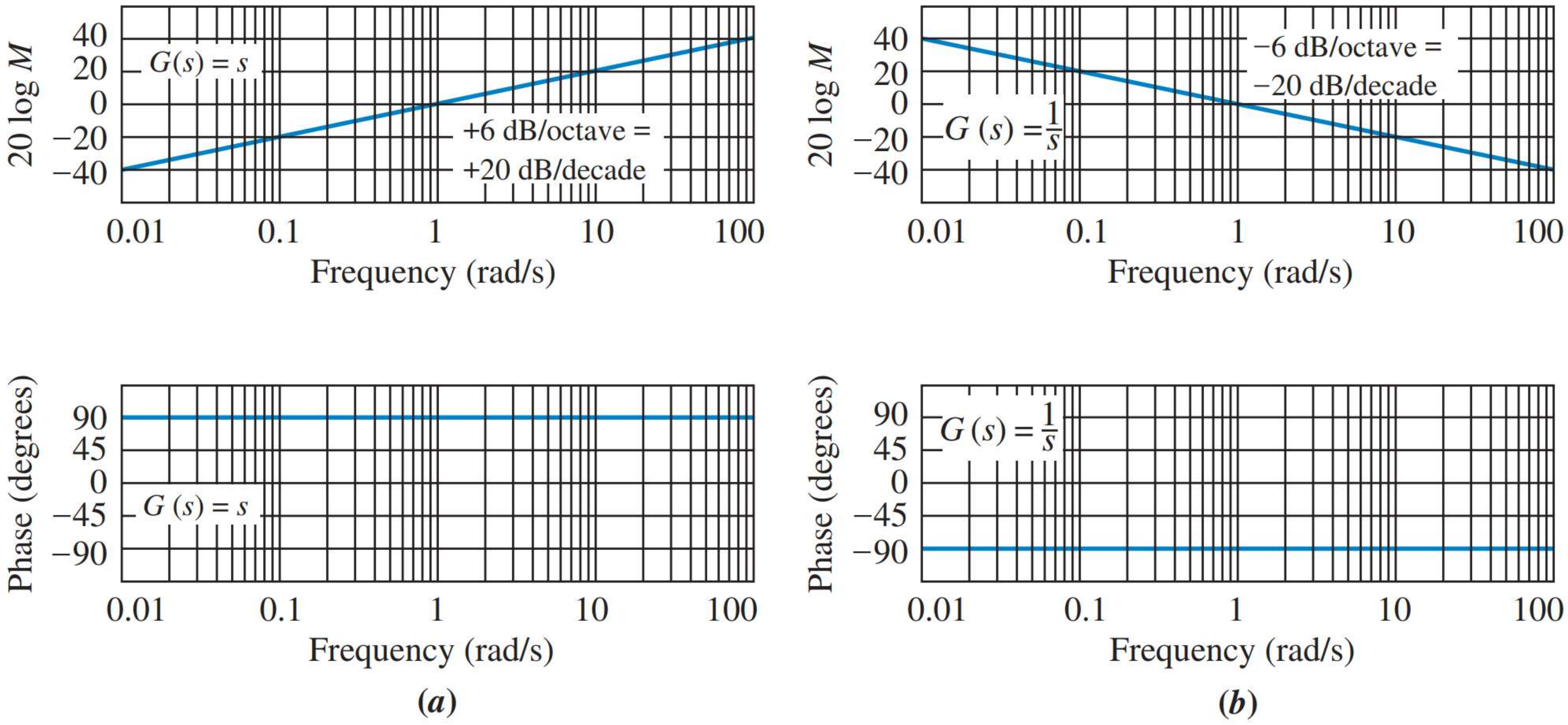

FIGURE 10.9 Normalized and scaled Bode plots for
a. $G(s) = s$;
b. $G(s) = 1/s$;
(*figure continues*)

Bode Plot for single zero and pole

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PROBLEM: Draw the Bode plots for the system shown in Figure 10.10, where $G(s) = K(s+3)/[s(s+1)(s+2)]$.

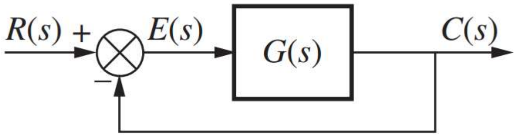
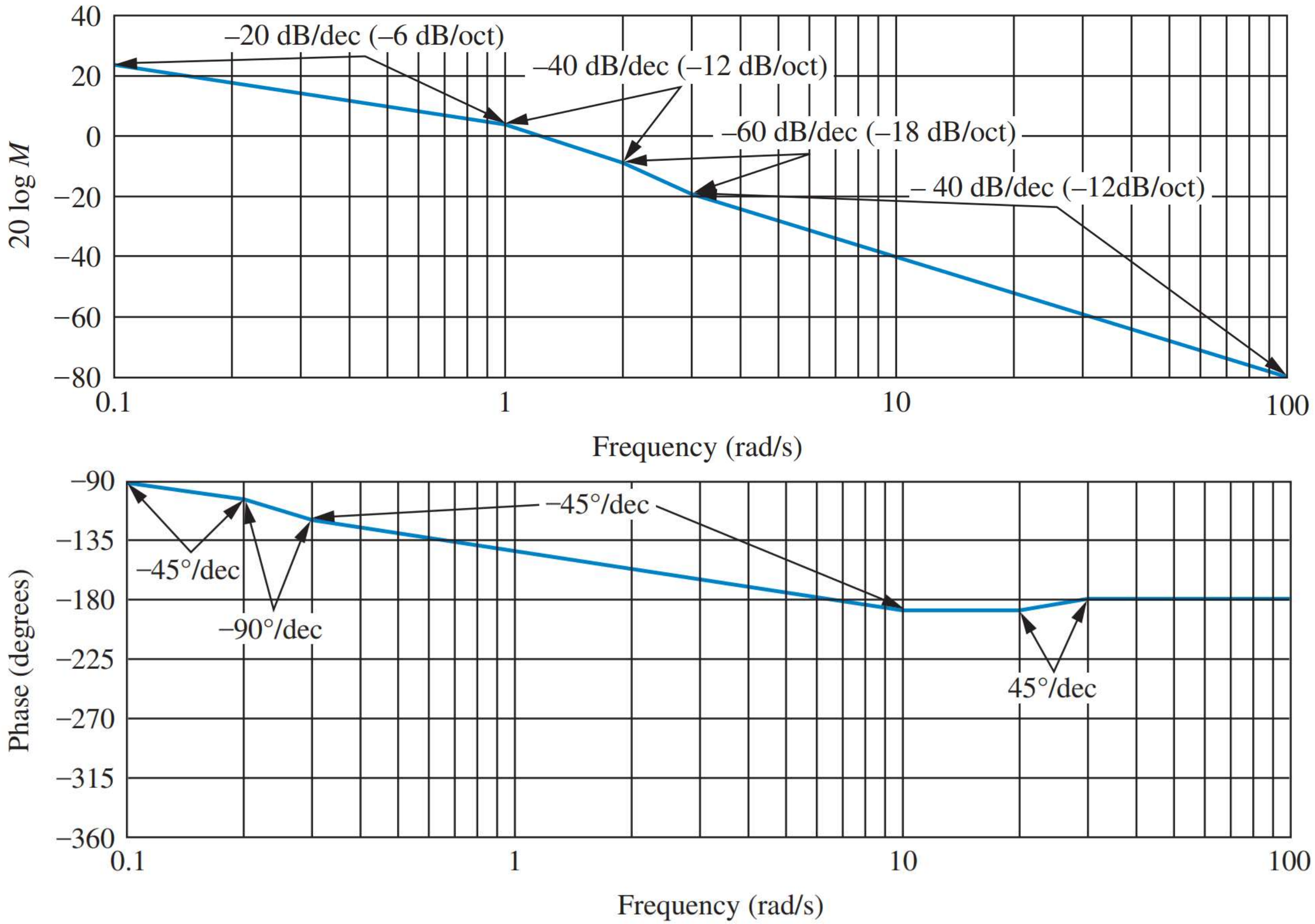


FIGURE 10.10 Closed-loop unity feedback system

$$G(s) = \frac{\frac{3}{2}K\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$



Bode Plot for single zero and pole

PROBLEM: Draw the Bode log-magnitude and phase plots for the system shown in Figure 10.10, where

$$G(s) = \frac{(s+20)}{(s+1)(s+7)(s+50)}$$

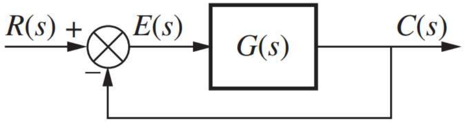


FIGURE 10.10 Closed-loop unity feedback system

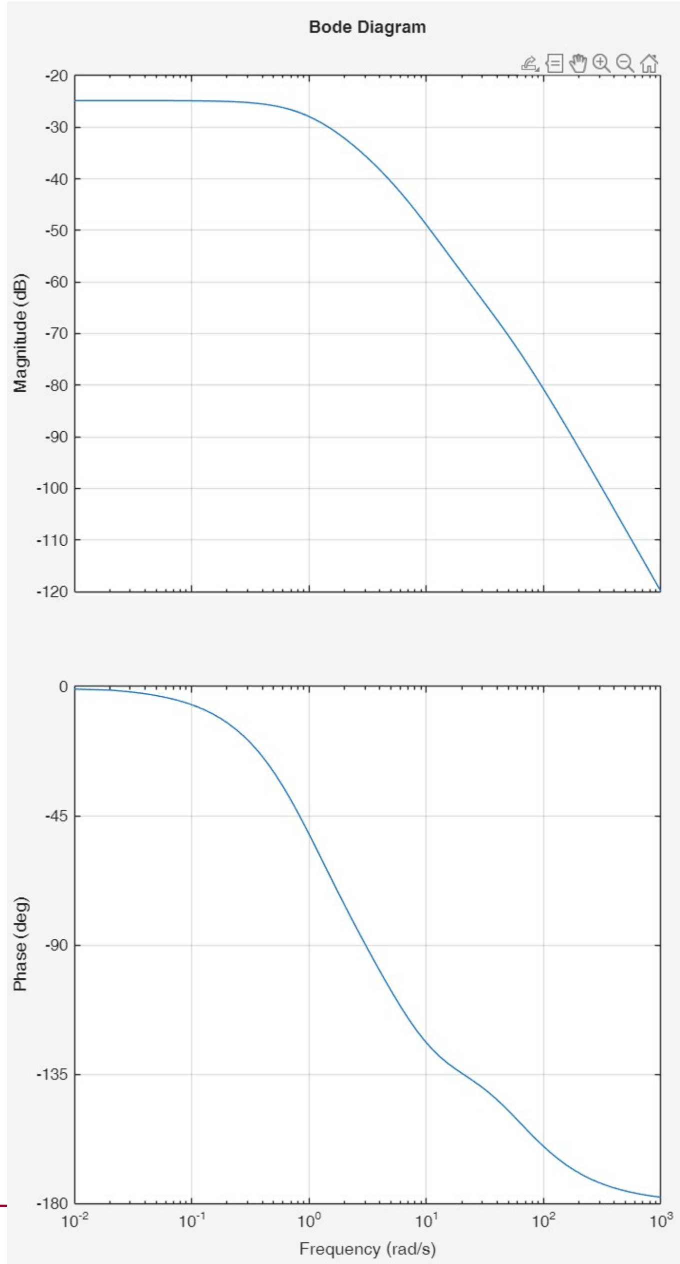
TryIt 10.1

Use MATLAB, the Control System Toolbox, and the following statements to obtain the Bode plots for the system of Skill-Assessment Exercise 10.2

```
G=zpk([-20],[-1, -7,...
  -50], 1)
bode (G); grid on
```

After the Bode plots appear, click on the curve and drag to read the coordinates.

```
>> clear all
>> s=tf('s')

s =

  s

Continuous-time transfer function.
Model Properties
>> G = (s+20)/(s+1)/(s+7)/(s+50)

G =

            s + 20
  --------------------------
  s^3 + 58 s^2 + 407 s + 350

Continuous-time transfer function.
Model Properties
>> bode(G)
>> |
```



Bode Plot for Poles of Conjugate Pair

Bode Plot for poles of conjugate pair

- For $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = \omega_n^2\left(\frac{s^2}{\omega_n^2} + 2\zeta\frac{s}{\omega_n} + 1\right)$
- The asymptotic approximation line has a slope of 40 dB/decade slot and phase shift of 180 degrees.

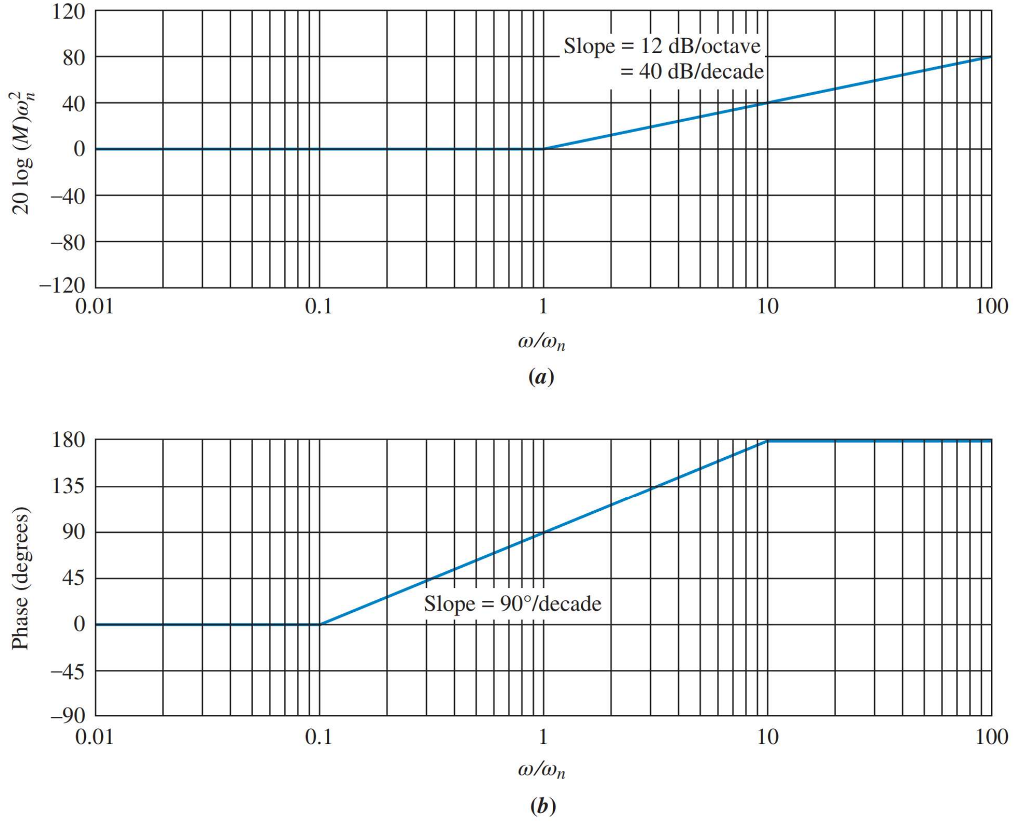


FIGURE 10.13 Bode asymptotes for normalized and scaled $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$: **a.** magnitude; **b.** phase

At low frequencies, $G(s) \approx \omega_n^2 = \omega_n^2\angle 0°$

At high frequencies, $G(s) \approx s^2$

Bode Plot for poles of conjugate pair

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- For $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2 = \omega_n^2\left(\frac{s^2}{\omega_n^2} + 2\zeta\frac{s}{\omega_n} + 1\right)$
- Unlike the first-order frequency response approximation, the difference between the asymptotic approximation and the actual frequency response can be great for some values of ζ.

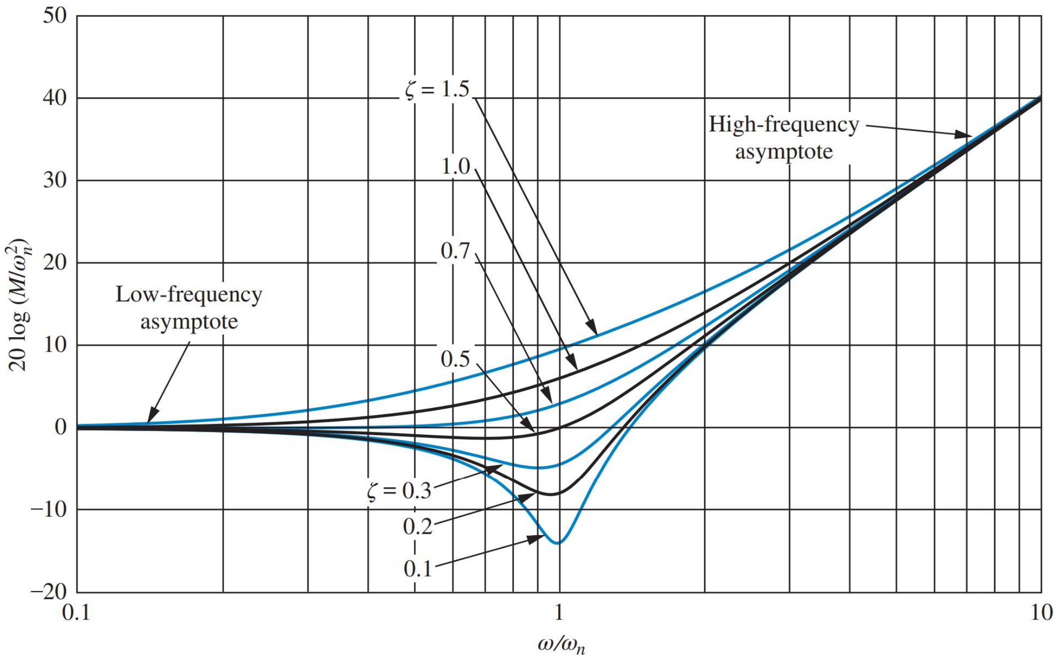

FIGURE 10.14 Normalized and scaled log-magnitude response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

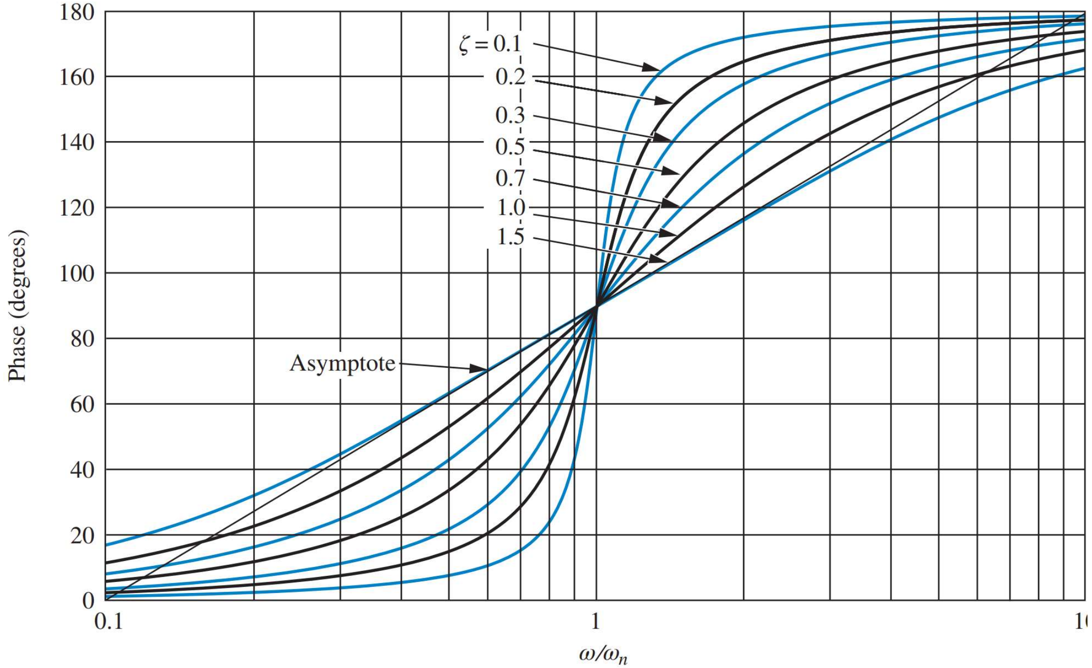

FIGURE 10.15 Scaled phase response for $(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Bode Plot for poles of conjugate pair

- It is also the case for its inverse of G(s):

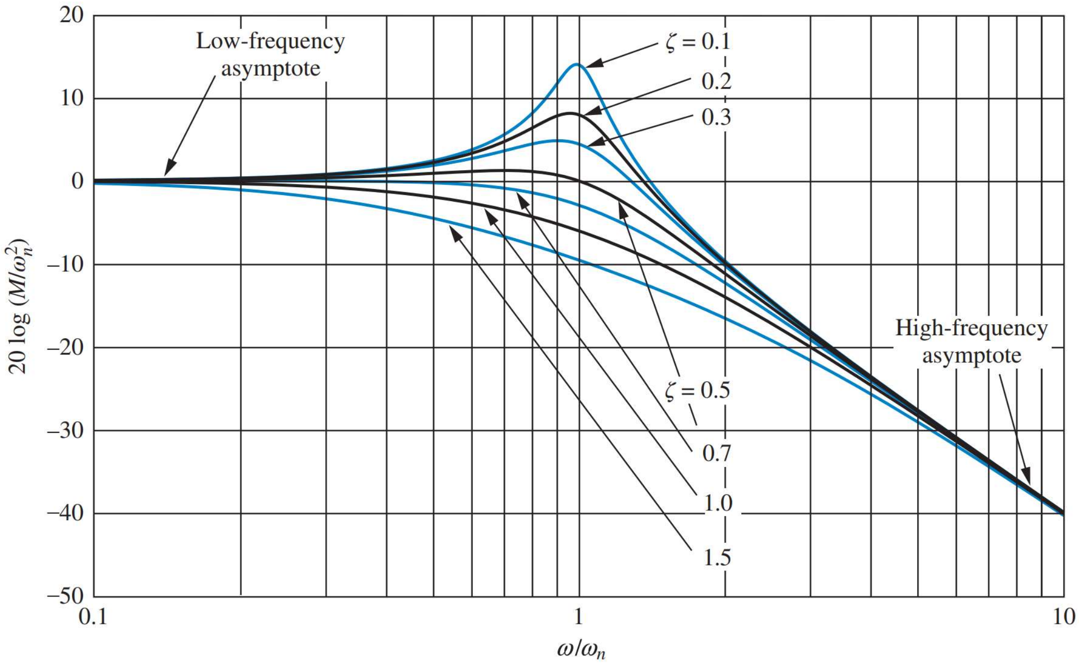

FIGURE 10.16 Normalized and scaled log-magnitude response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

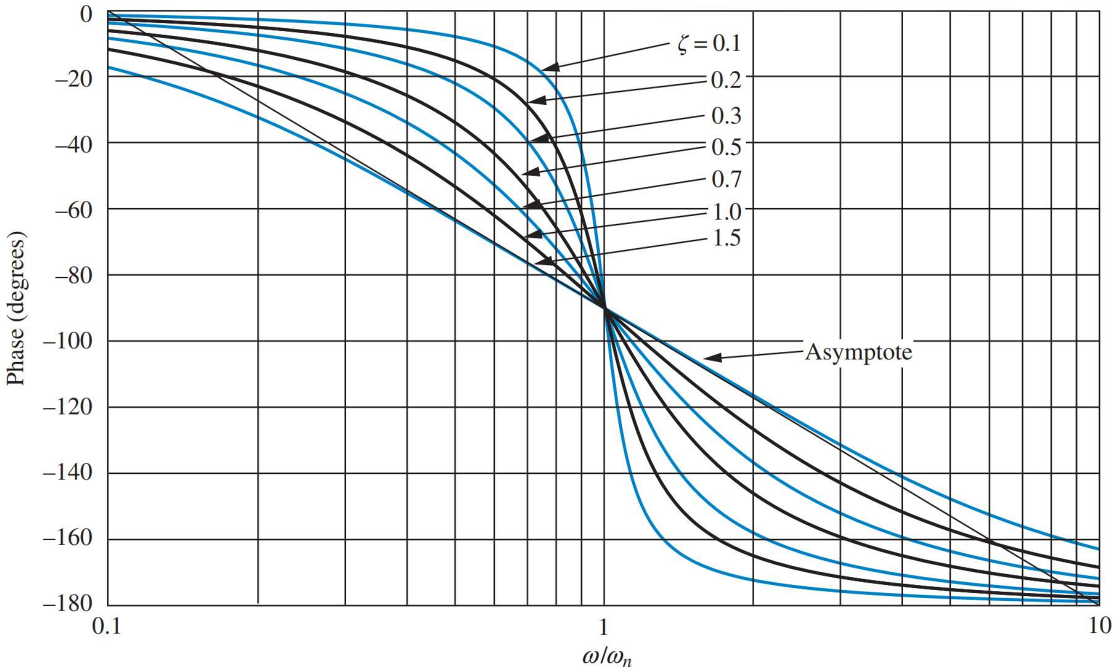

FIGURE 10.17 Scaled phase response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$

Stability Margin In Terms of Bode Plot

Stability Margin in terms of F.R.

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- When we are discussing stability, we are implying the stability of the closed loop control system.
- The frequency response of a closed loop transfer function can be derived as follows.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$\Rightarrow T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)}$$

- Note this $T(j\omega)$ is the complex gain to a sinusoidal input $R(j\omega)$ and **apparently this gain should not be equal to infinity (as a sufficient condition),** that is, the open loop transfer function should satisfy:

$$\Rightarrow L(j\omega) \triangleq G(j\omega)H(j\omega) \neq -1 = 1\angle 180^\circ$$

Gain margin, G_M. The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180° of phase shift to make the closed-loop system unstable.

Phase margin, Φ_M. The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

Stability Margin in terms of F.R.

Gain margin, G_M. The gain margin is the change in open-loop gain, expressed in decibels (dB), required at 180° of phase shift to make the closed-loop system unstable.

Phase margin, Φ_M. The phase margin is the change in open-loop phase shift required at unity gain to make the closed-loop system unstable.

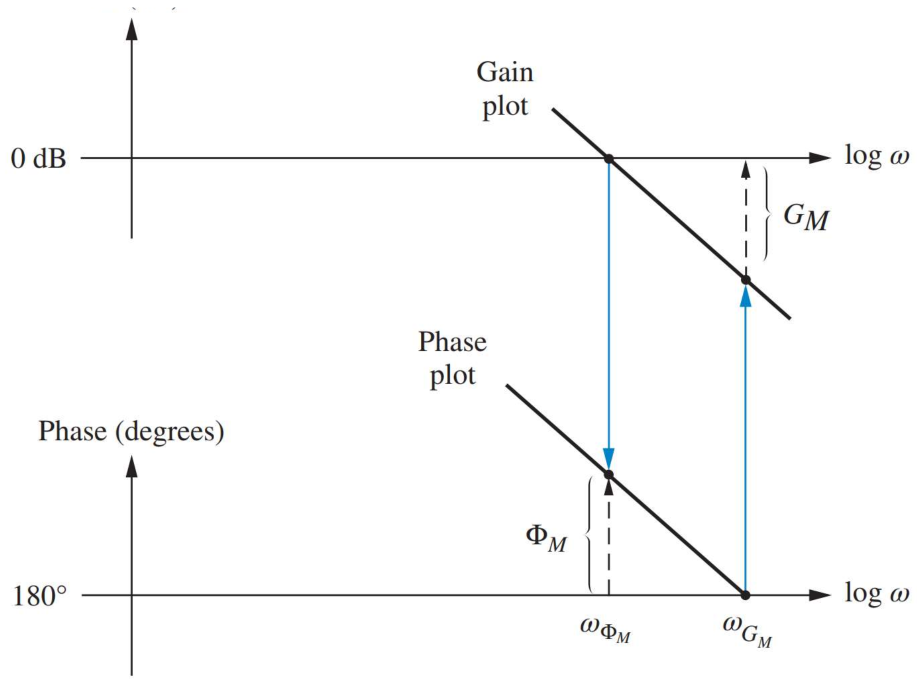

FIGURE 10.37 Gain and phase margins on the Bode diagrams

Stability Margin in terms of F.R.

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PROBLEM: Use Bode plots to determine the range of K within which the unity feedback system shown in Figure 10.10 is stable. Let $G(s) = K/[(s+2)(s+4)(s+5)]$.

PROBLEM: If $K = 200$ in the system of Example 10.9, find the gain margin and the phase margin.

SOLUTION: The Bode plot in Figure 10.36 is scaled to a gain of 40. If $K = 200$ (five times as great), the magnitude plot would be $20 \log 5 = 13.98$ dB higher.

To find the gain margin, look at the phase plot and find the frequency where the phase is 180°. At this frequency, determine from the magnitude plot how much the gain can be increased before reaching 0 dB. In Figure 10.36, the phase angle is 180° at approximately 7 rad/s. On the magnitude plot, the gain is $-20 + 13.98 = -6.02$ dB. Thus, the gain margin is 6.02 dB.

To find the phase margin, we look on the magnitude plot for the frequency where the gain is 0 dB. At this frequency, we look on the phase plot to find the difference between the phase and 180°. This difference is the phase margin. Again, remembering that the magnitude plot of Figure 10.36 is 13.98 dB lower than the actual plot, the 0 dB crossing (−13.98 dB for the normalized plot shown in Figure 10.36) occurs at 5.5 rad/s. At this frequency the phase angle is −165°. Thus, the phase margin is $-165° - (-180°) = 15°$.

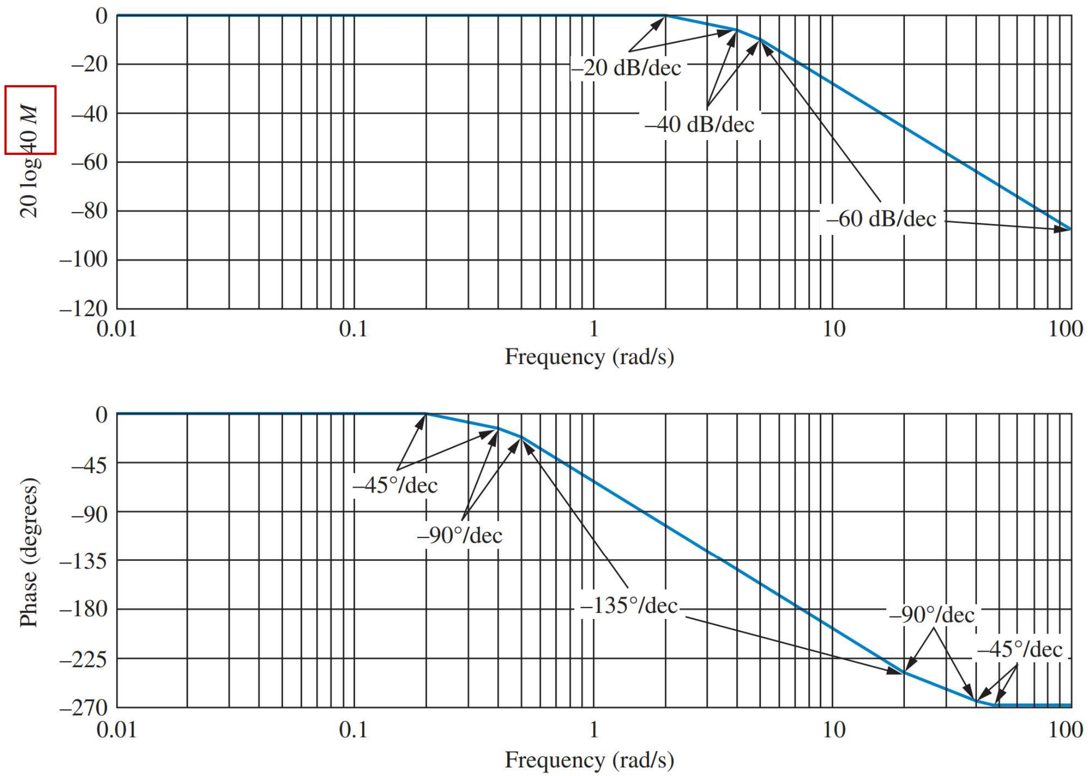


FIGURE 10.36 Bode log-magnitude and phase diagrams for the system of Example 10.9

Stability Margin in terms of F.R.

PROBLEM: For the system shown in Figure 10.10, where

$$G(s) = \frac{K}{(s+5)(s+20)(s+50)}$$

do the following:

a. Draw the Bode log-magnitude and phase plots.

b. Find the range of K for stability from your Bode plots.

c. Evaluate gain margin, phase margin, zero dB frequency, and 180° frequency from your Bode plots for $K = 10{,}000$.

ANSWERS:

TryIt 10.4

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 10.6(c) using Bode plots.

```
G=zpk([],...
 [-5, -20, -50], 10000)
bode(G)
grid on
```

After the Bode plot appears:

1. Right-click in the graph area.
2. Select **Characteristics**.
3. Select **All Stability Margins**.
4. Let the mouse rest on the margin points to read the gain and phase margins.

Compensator Design based off Bode Plot

Why?

- Root locus requires repeated trials to find the desired design point from which the gain can be obtained.
  - For example, in designing gain to meet a percent overshoot requirement, root locus requires the search of a radial line for the point where the open-loop transfer function yields an angle of 180°.
  - To evaluate the range of gain for stability, root locus requires a search of the jω-axis for 180°.

- On the other hand, the F.R. method can be extended to multiple cascaded compensators, and the compensator parameters can be explicitly calculated with simple scripts.

Translate F.R. Peak into Damping Ratio (not useful)

- Consider the closed loop frequency response of a second order system:

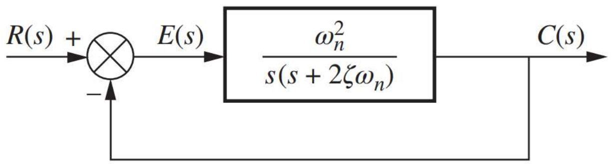


FIGURE 10.38 Second-order closed-loop system

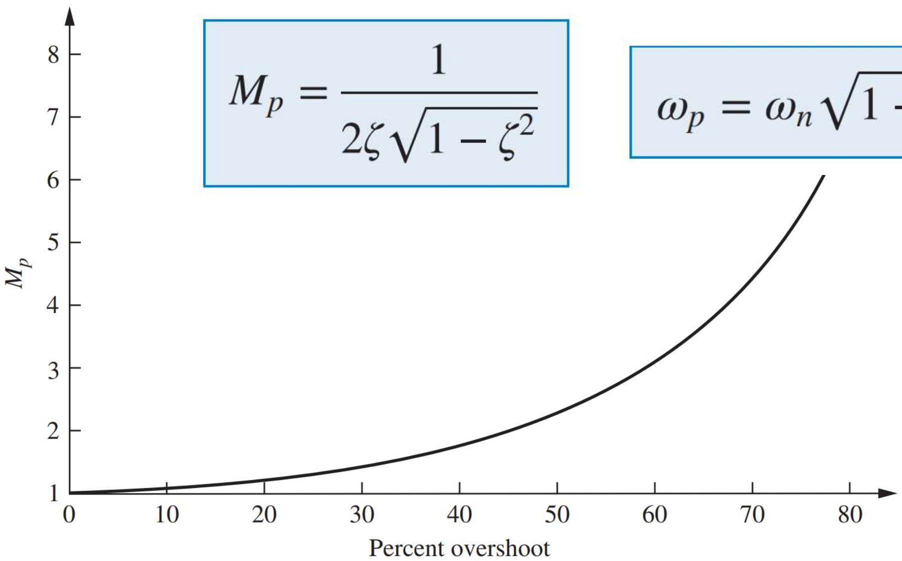
$$\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

- Differentiating w.r.t. ω^2 yields peak information:

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$



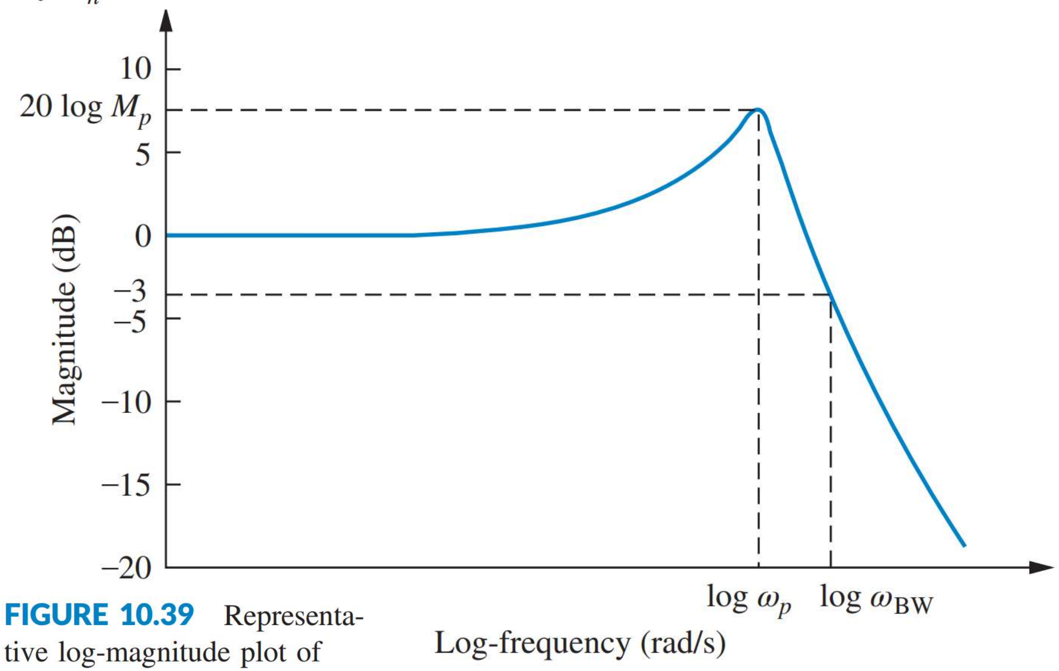


FIGURE 10.39 Representative log-magnitude plot of Eq. (10.51)

Translate Bandwidth into Time Domain (not useful)

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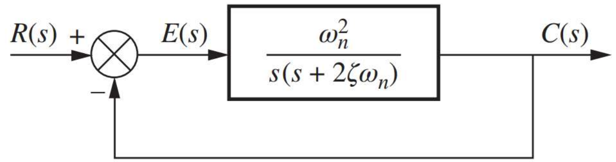
- The bandwidth of a two-pole system can be found by finding that frequency for which $M = \frac{1}{\sqrt{2}} = -3\text{dB}$.

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

$$\omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$\omega_n = 4/T_s\zeta$$

$$\omega_{BW} = \frac{4}{T_s\zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$



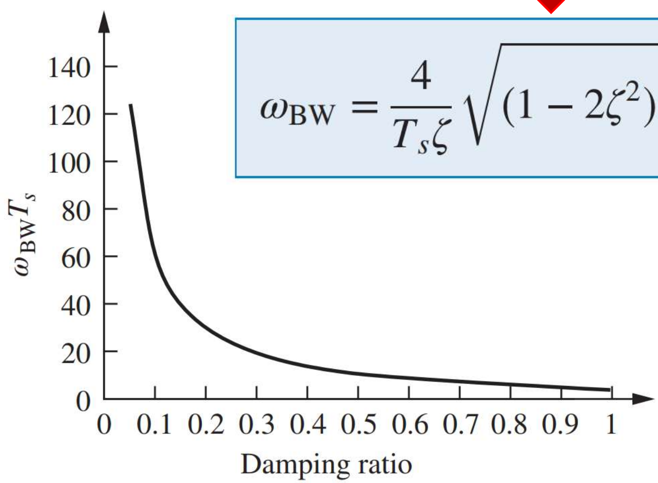


FIGURE 10.41 Normalized bandwidth vs. damping ratio for ... settling time;

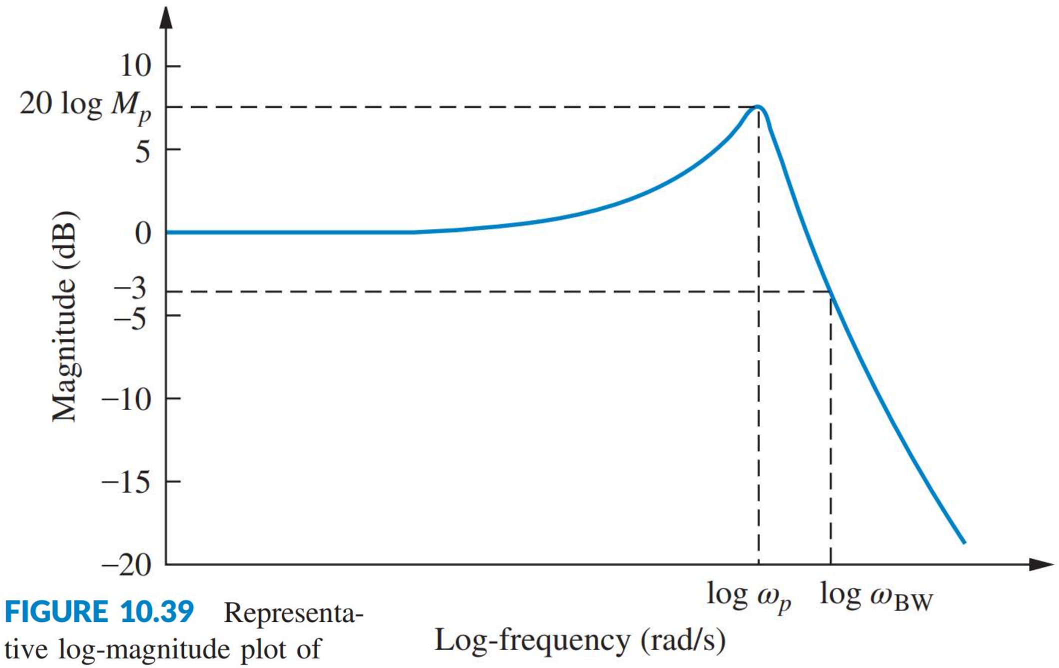


FIGURE 10.39 Representative log-magnitude plot of Eq. (10.51)

Translate Phase Marin into Damping Ratio (useful)

- We can derive the relation between the phase margin Φ_M and the damping ratio ζ for a standard second order system:

$$\Phi_M = 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}{2\zeta} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

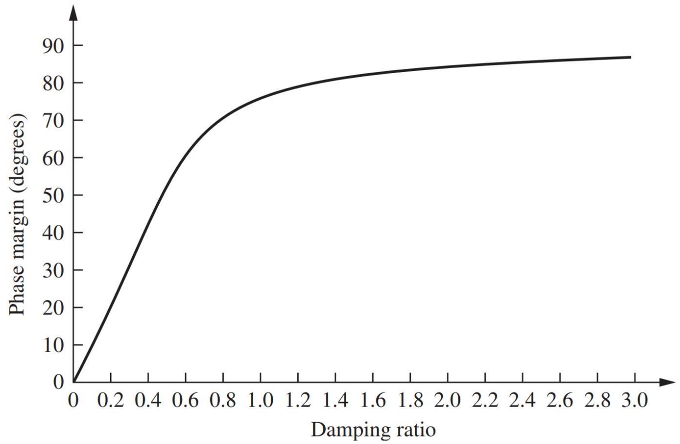


FIGURE 10.48 Phase margin vs. damping ratio

- Details:

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$|G(j\omega)| = \frac{\omega_n^2}{|-\omega^2 + j2\zeta\omega_n\omega|} = 1$$

$$\omega_1 = \omega_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}$$

Phase F.R. →

$$\angle G(j\omega) = -90 - \tan^{-1}\frac{\omega_1}{2\zeta\omega_n} = -90 - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}$$

F.R. based Design with Only Gain Adjustment

Transient Response Design via Gain Adjustment

PROBLEM: For the position control system shown in Figure 11.2, find the value of preamplifier gain, K, to yield a 9.5% overshoot in the transient response for a step input. Use only frequency response methods.

$$\Phi_M = 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}{2\zeta} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} \quad (10.73)$$

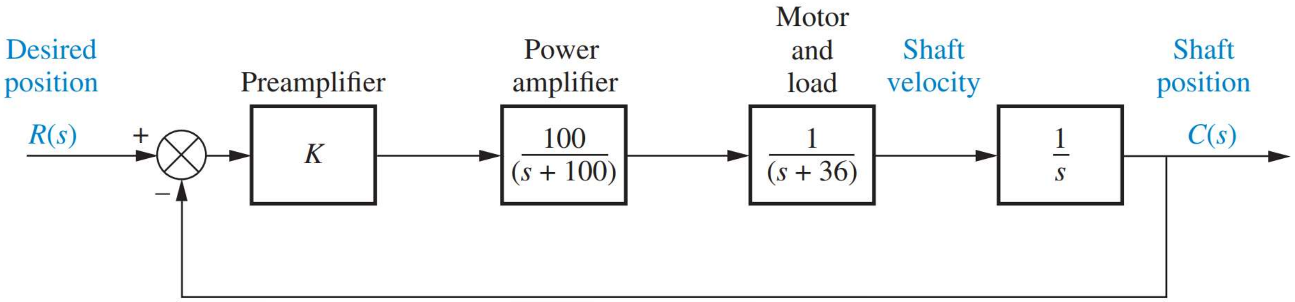


FIGURE 11.2 System for Example 11.1

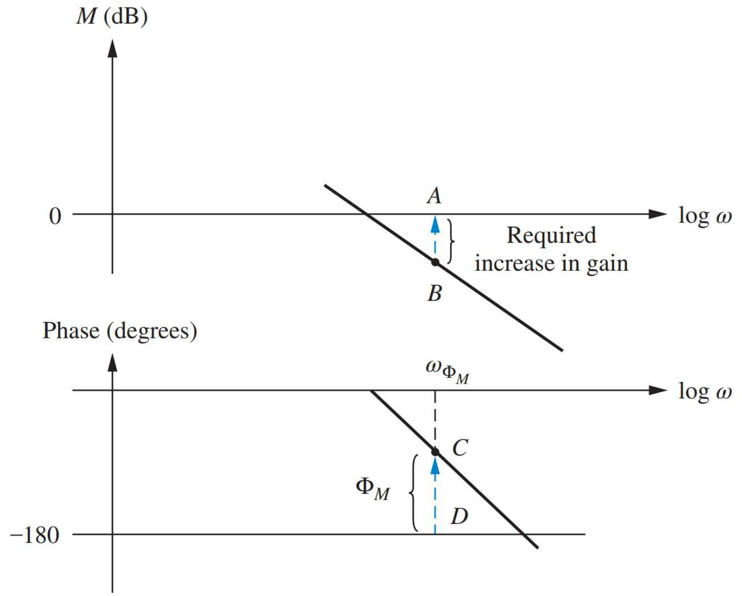


FIGURE 11.1 Bode plots showing gain adjustment for a desired phase margin

SOLUTION:

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)}$$

F.R. based Design with Only Gain Adjustment

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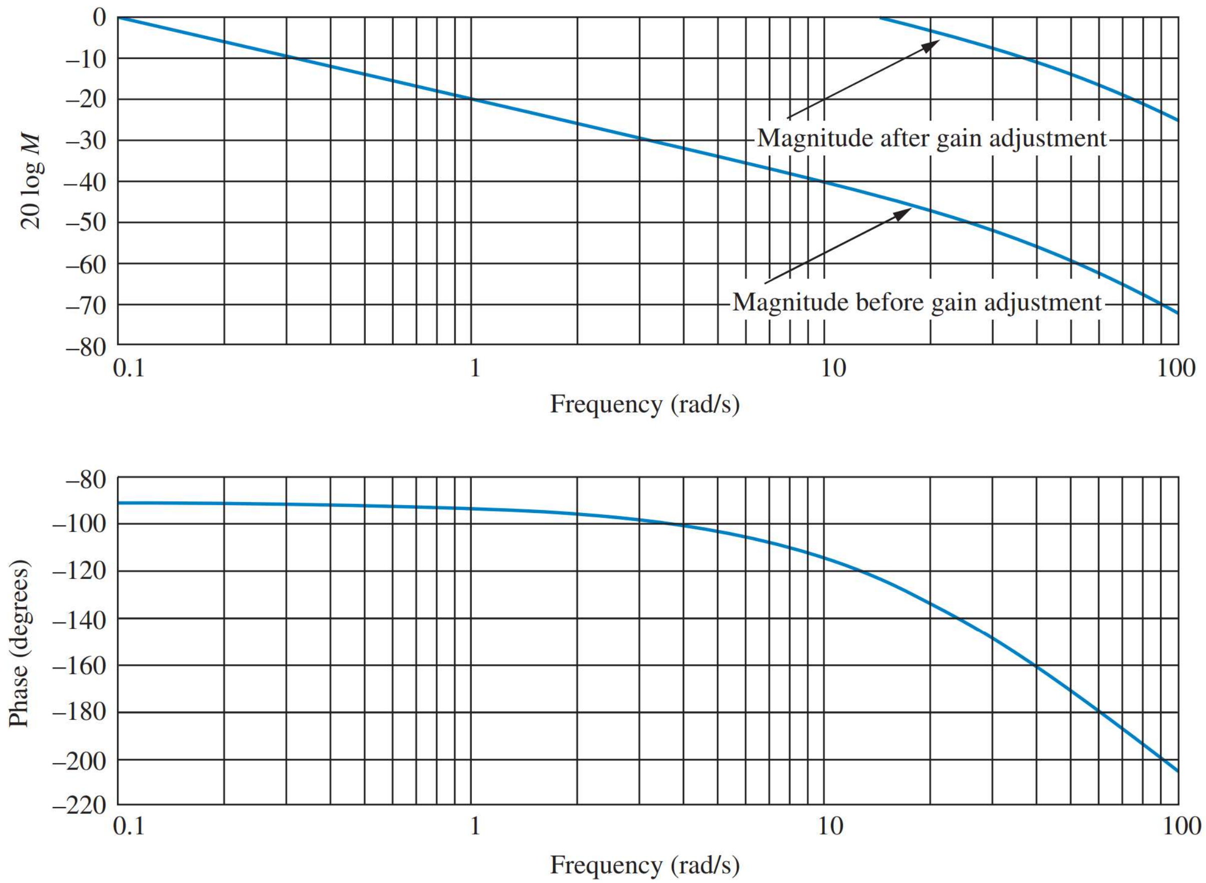

FIGURE 11.3 Bode magnitude and phase plots for Example 11.1

TABLE 11.1 Characteristic of gain-compensated system of Example 11.1

Parameter	Proposed specification	Actual value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

- Note phase margin and percent overshoot are due to **damping ratio requirement**.
- Other potential **requirement** would include:
  - steady state error constant K_v
    - Need lag compensation
  - and cut-off frequency (phase-margin frequency) ω_c
    - Need gain adjustment and in this case the phase margin requirement can not be met, which implies the need of lead compensation.

F.R. of Lead Compensator

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- Frequency response of lead compensator can add phase margin to the original system and at the same time **the cut-off frequency ω_c is increased**.

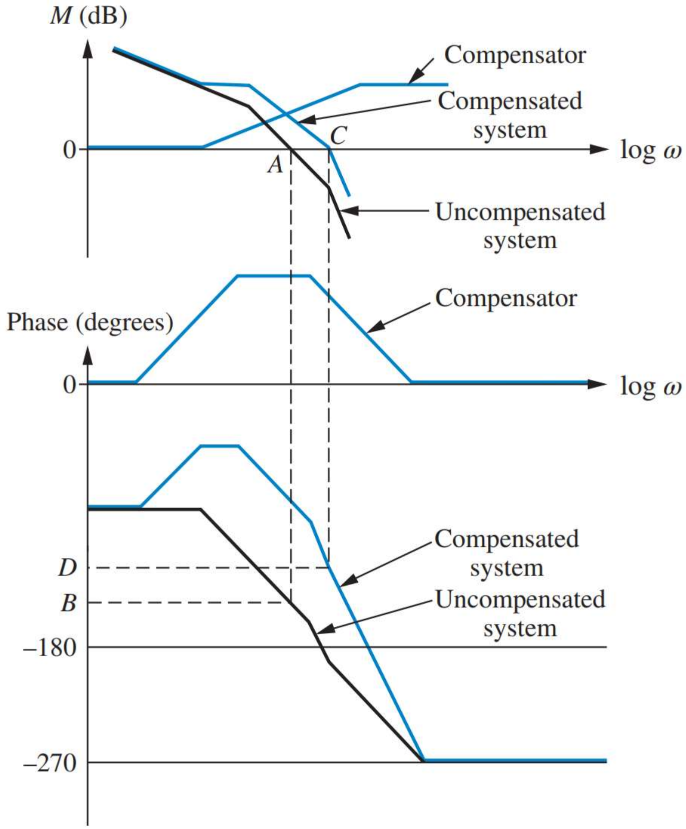

FIGURE 11.7 Visualizing lead compensation

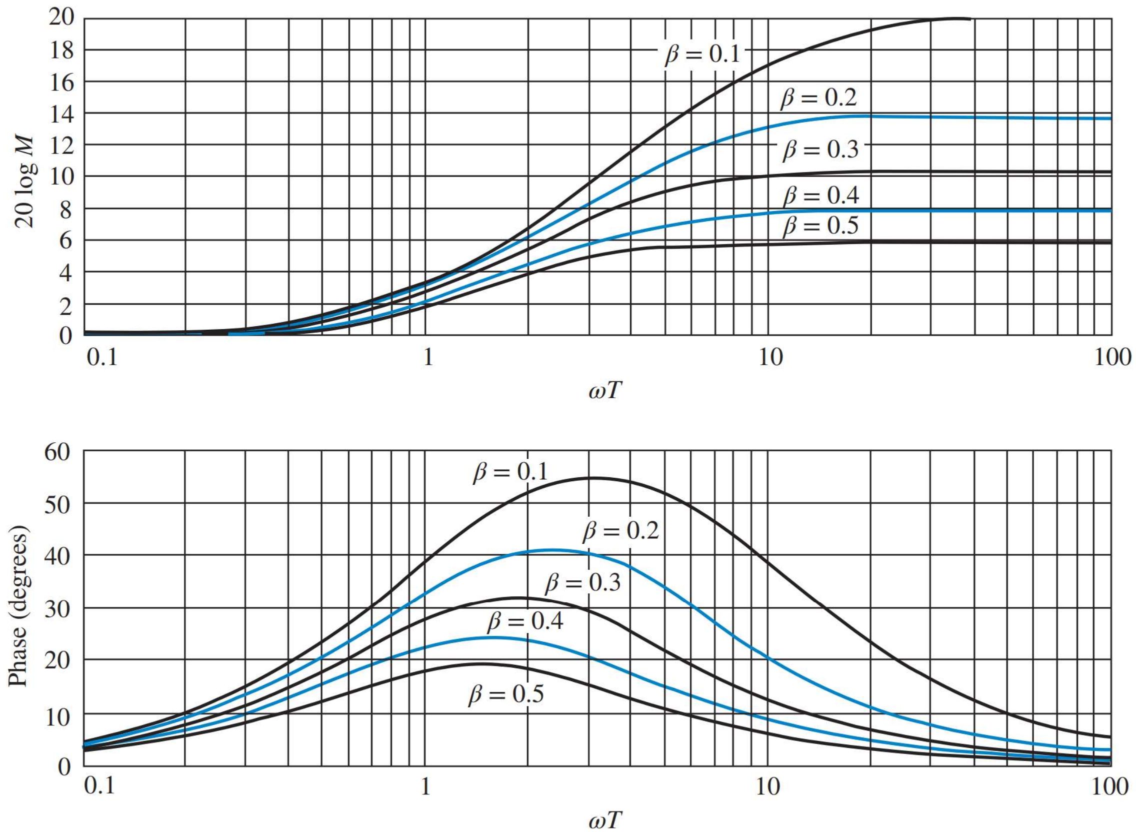

FIGURE 11.8 Frequency response of a lead compensator, $G_c(s) = [1/\beta][(s + 1/T)/(s + 1/\beta T)]$

F.R. of Lag Compensator

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- Frequency response of lag compensator:
  - It is possible to introduce a lag compensator to **decrease** **cut-off frequency $\boldsymbol{\omega_c}$** so as to exploit the original system's phase margin

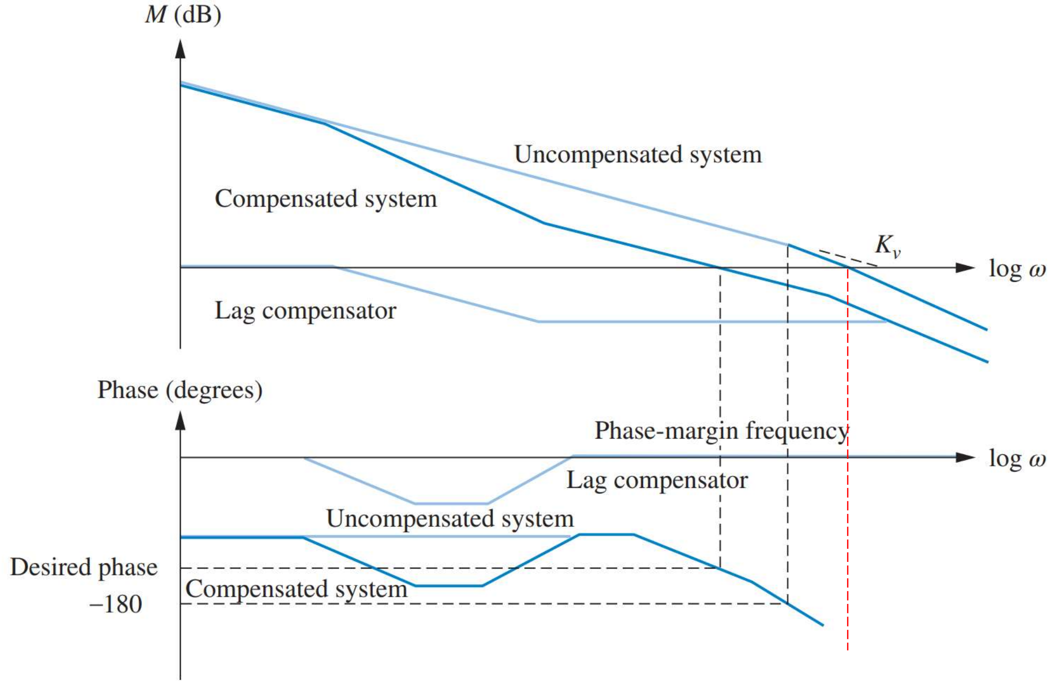
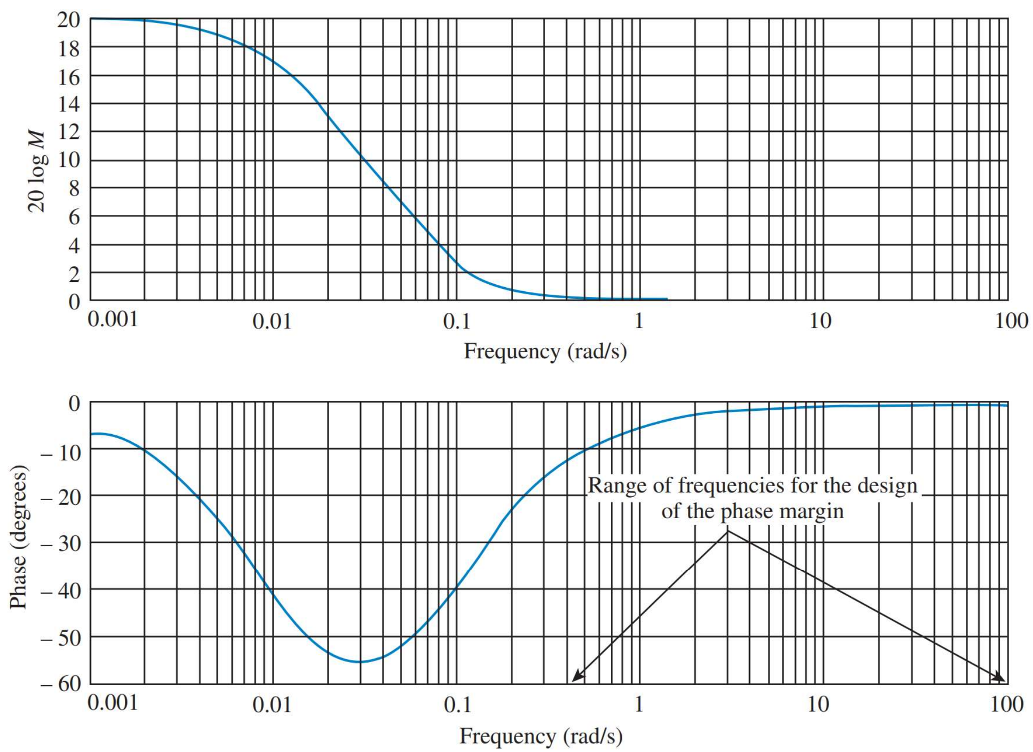


FIGURE 11.4 Visualizing lag compensation

FIGURE 11.5 Frequency response plots of a lag compensator, $G_c(s) = (s + 0.1)/(s + 0.01)$



Lag-Lead Compensation Design

PROBLEM: Given a unity feedback system where $G(s) = K/[s(s+1)(s+4)]$, design a passive lag-lead compensator using Bode diagrams to yield a 13.25% overshoot, a peak time of 2 seconds, and $K_v = 12$.

SOLUTION: We will follow the steps previously mentioned in this section for lag-lead design.

1. The bandwidth required for a 2-seconds peak time is 2.29 rad/s.
2. In order to meet the steady-state error requirement, $K_v = 12$, the value of K is 48.
3. The Bode plots for the uncompensated system with $K = 48$ are shown in Figure 11.12. We can see that the system is unstable.
4. The required phase margin to yield a 13.25% overshoot is 55°.
5. Let us select $\omega = 1.8$ rad/s as the new phase-margin frequency.
6. At this frequency, the uncompensated phase is $-176°$ and would require, if we add a $-5°$ contribution from the lag compensator, a 56° contribution from the lead portion of the compensator.

……

NOT RECOMMENDED!

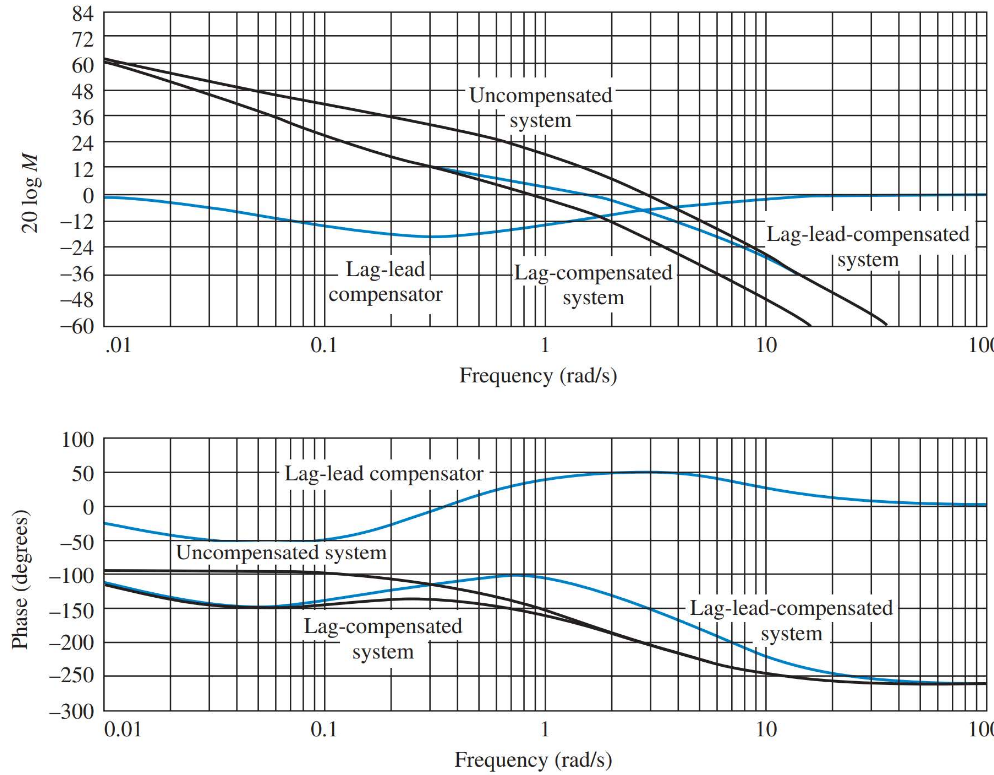
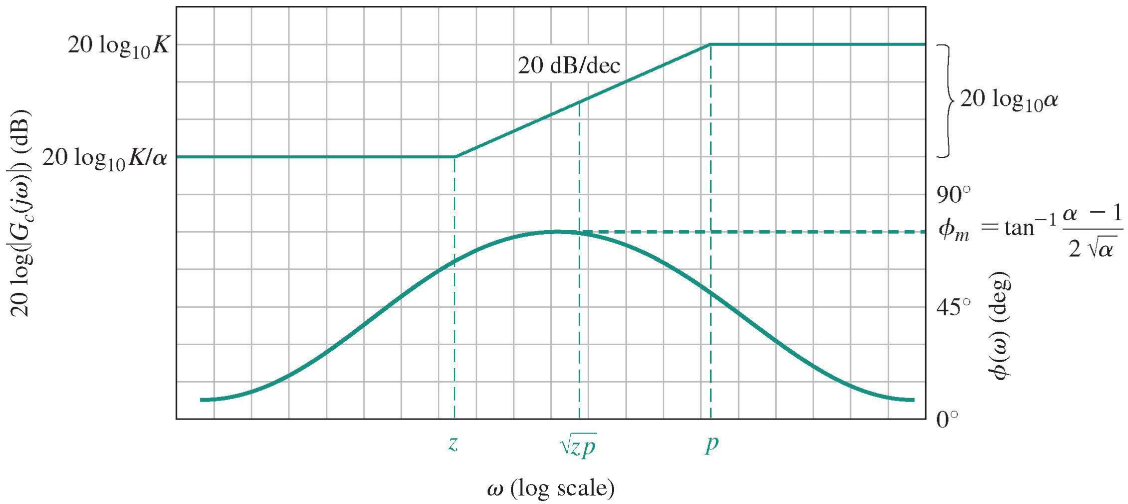


FIGURE 11.12 Bode plots for lag-lead compensation in Example 11.4

My Personally Recommended Standard Procedure

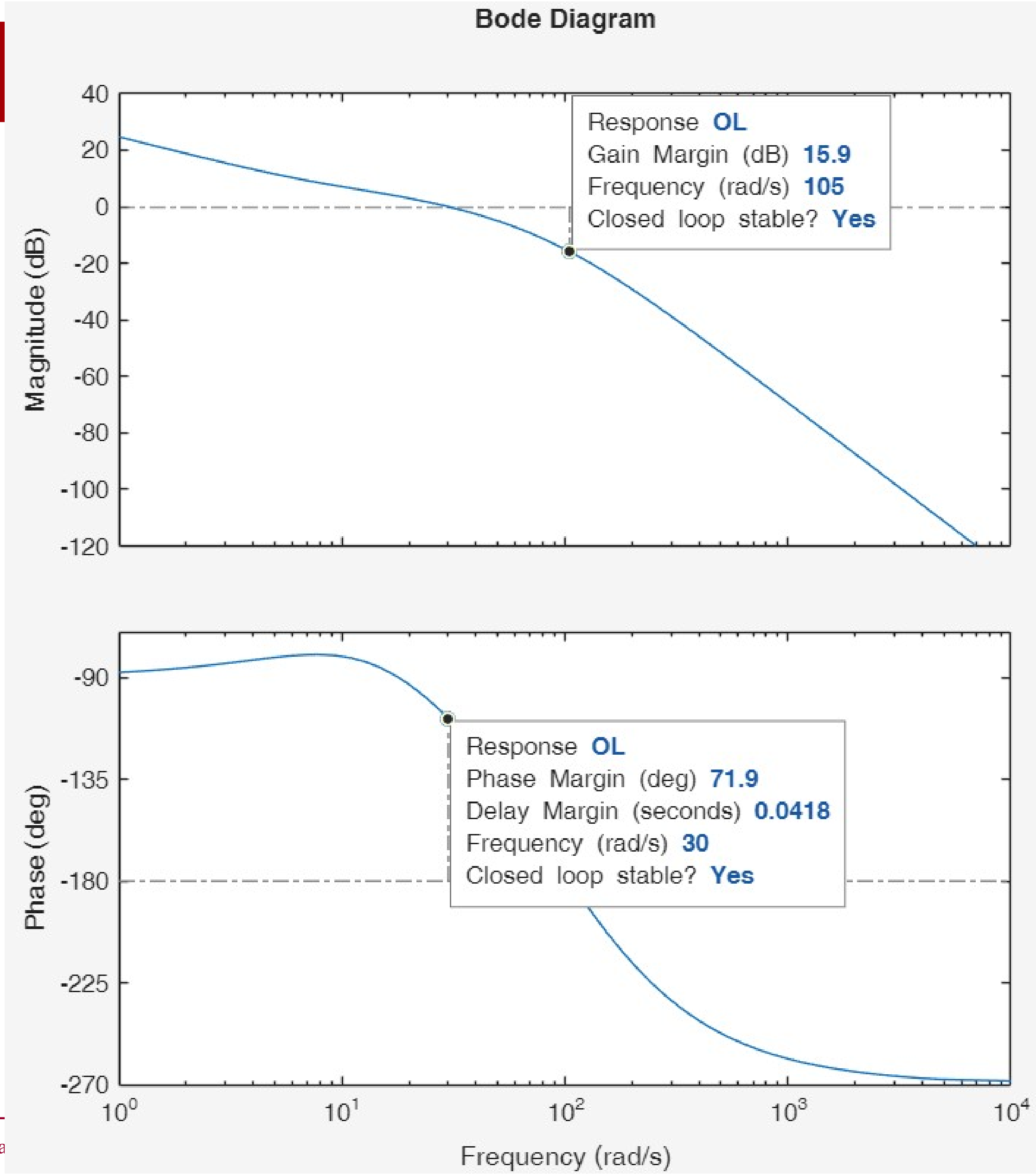
- Consider a unity negative feedback system with a plant $P(s) = \frac{1}{s}\frac{1}{0.1s+1}\frac{1}{0.05s+1}$.
  - Try to design cascade compensator such that the **velocity constant** is larger than 20, the **corner frequency** is at least 30 rad/s, and the **phase margin** is at least 50°.
  - Hint: if you find one lead compensator is not enough, consider adding one more.
- The lag compensator for steady state error is designed after the phase margin requirement has been met.




```
close all;clc;clear all;
s = tf('s');
P = 1 / (s) / (0.05*s+1) / (0.1*s+1);
wc = 30;
K = 10^(-10/20) / abs(evalfr(P, wc*1j))
figure();
bode(K*P)
wm = wc;
alpha = 10;
z1 = sqrt(wm^2/alpha);
p1 = alpha * z1;
C = (s/z1+1) / (s/p1+1);
figure();
bode(C)
OL = C*K*P;
figure();
bode(OL)
allmargin(OL)
```

The maximum value of the phase lead occurs at:

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$




```
close all;clc;clear all;
s = tf('s');
P = 1 / (s) / (0.05*s+1) / (0.1*s+1);
wc = 30;
K = 10^(-10/20) / abs(evalfr(P, wc*1j))
figure();
bode(K*P)
wm = wc;
alpha = 10;
z1 = sqrt(wm^2/alpha);
p1 = alpha * z1;
C = (s/z1+1) / (s/p1+1);
figure();
bode(C)
OL = C*K*P;
figure();
bode(OL)
allmargin(OL)
```

The maximum value of the phase lead occurs at:

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

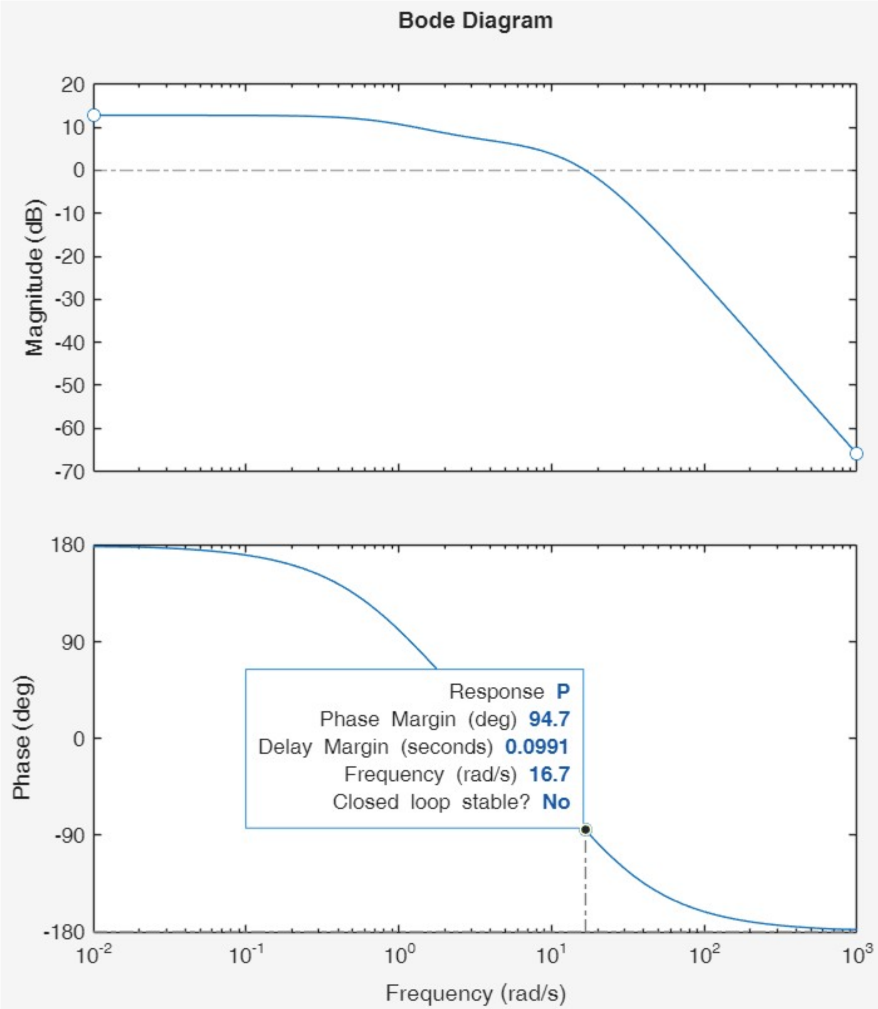
Non-minimum Phase System

(for which the stability margins are no longer valid)

Nonminimum Phase System due to RHP Zero/Pole

- As a motivation, consider a transfer function, and if its open loop pole or zero is mirrored about the $j\omega$-axis, i.e., it is moved to the right hand s-plane, the new transfer function shares the same **magnitude** frequency response but its **phase** frequency response would become more lagging or advancing over frequency variable ω.
- An example is provided as script below. Can you check its stability and if it is unstable can you make it stable?

```
close all; cla; clc
s = zpk(0, [], 1);
Popen = 500 * (s+2)/(s+1)/(s^2+30*s+229)
Popen = 500 * (s-2)/(s+1)/(s^2+30*s+229)
P = Popen
allmargin(Popen)
h = bodeplot(Popen);
h.showCharacteristic('AllStabilityMargins')
step(Popen)
P = Popen/(1+Popen); step(P)
```



Nonminimum Phase System due to Time Delay

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- **Time delay**, often due to the non-responsive time of an actuator, is a typical component that makes a nonminimum phase system:
  - Time domain: $\delta(t-T_d)$
  - Frequency domain: e^{-sT_d}

  whose Bode plot is not yet learned.

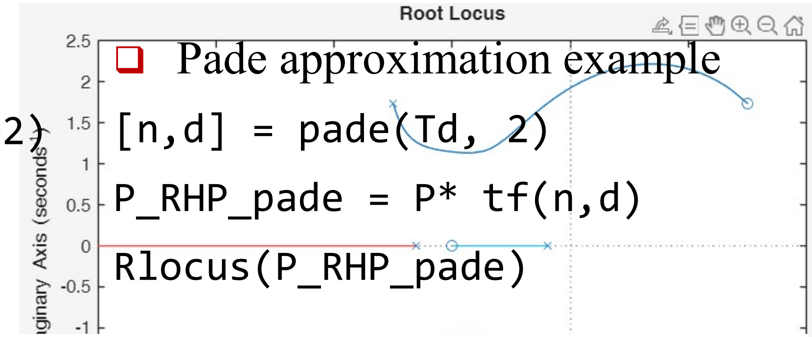
TABLE 2.2 Laplace transform theorems

Item no.	Theorem
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$
2.	$\mathscr{L}[kf(t)] = kF(s)$
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$

```
>> [n,d] = pade(Td, 2)

n =

     1    -6    12


d =

     1     6    12
```

- Root locus fails to visualize time delay. The script on the right gives: *Error using DynamicSystem/rlocus The "rlocus" command cannot be used for continuous-time models with delays. Use the "pade" command to approximate delays."*

```
s = zpk(0, [], 1);
L = exp(-s)/(s+1)/(s+2)
rlocus(L)
```



- Pade approximation example

```
[n,d] = pade(Td, 2)
P_RHP_pade = P* tf(n,d)
Rlocus(P_RHP_pade)
```

- Pade approximation uses Taylor series $e^{-sT_d} = e^{-x}$

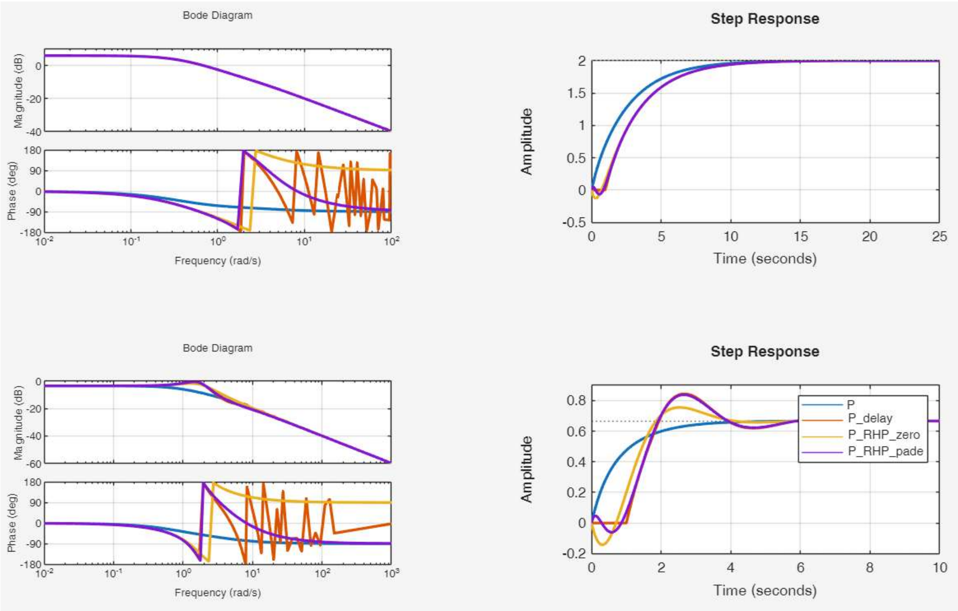
$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots$$

A portion of the Padé table for the exponential function e^z

m \ n	0	1	2	
0	$\frac{1}{1}$	$\frac{1}{1-z}$	$\frac{1}{1-z+\frac{1}{2}z^2}$	$1-z$…
1	$\frac{1+z}{1}$	$\frac{1+\frac{1}{2}z}{1-\frac{1}{2}z}$	$\frac{1+\frac{1}{3}z}{1-\frac{2}{3}z+\frac{1}{6}z^2}$	$1-\frac{3}{4}$…
2	$\frac{1+z+\frac{1}{2}z^2}{1}$	$\frac{1+\frac{2}{3}z+\frac{1}{6}z^2}{1-\frac{1}{3}z}$	$\frac{1+\frac{1}{2}z+\frac{1}{12}z^2}{1-\frac{1}{2}z+\frac{1}{12}z^2}$	$1+$… / $1-\frac{3}{5}$…

Nonminimum Phase System due to Time Delay

- **Time delay** example and comparison of the **four** systems having the same open loop magnitude frequency response:
  - P
  - P_delay
  - P_RHP_zero
  - P_RHP_pade




```
%% Delay and nonminimum phase system
Td = 1.0; % time delay = 1 sec
P = tf([1,2], [1,3,1])
P_delay = tf([1,2], [1,3,1], 'InputDelay', Td)
P_RHP_zero = tf([-1,2], [1,3,1])
[n,d] = pade(Td, 2)
P_RHP_pade = P* tf(n,d)
options = bodeoptions;
options.PhaseWrapping = 'on';
subplot(221); bode(P, P_delay, P_RHP_zero, P_RHP_pade, options); grid; h1 =
findobj(gcf,'type','line'); set(h1,'linewidth',2);
subplot(222); step(P, P_delay, P_RHP_zero, P_RHP_pade); grid; h1 =
findobj(gcf,'type','line'); set(h1,'linewidth',2);
% closed loop response
P = P/(1+P)
P_delay = P_delay /(1+P_delay)
P_RHP_zero = P_RHP_zero /(1+P_RHP_zero )
P_RHP_pade = P_RHP_pade /(1+P_RHP_pade )
subplot(223); bode(P, P_delay, P_RHP_zero, P_RHP_pade, options); grid; h1 =
findobj(gcf,'type','line'); set(h1,'linewidth',2);
subplot(224); step(P, P_delay, P_RHP_zero, P_RHP_pade); grid; h1
=findobj(gcf,'type','line'); set(h1,'linewidth',2);
legend
```

Practical Minor and Major Loop design in Frequency Domain

- The speed regulation of a servo system is typically equipped with current sensor and an encoder for reconstructing angular velocity Ω as well as the torque-producing current i_q.
  - We use two series PI regulators to regulate the two feedback signals, but the problem is how to tune four parameters?

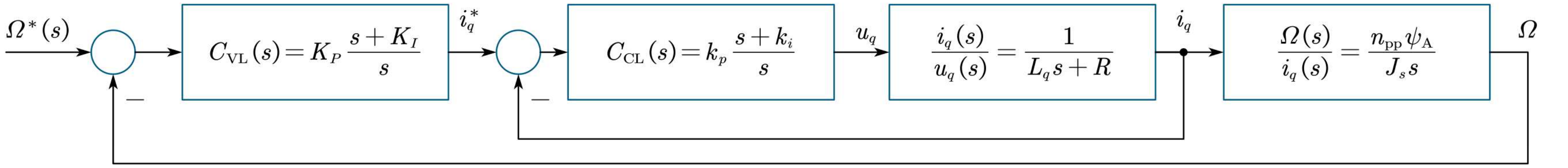


Figure 35. The nested loop controller for field oriented control of ac motor. The Park transform is omitted.

$$\frac{i_q}{i_q^*} = \frac{1}{\frac{s}{\mathrm{CLBW}} + 1} \qquad k_p = L_q \times \mathrm{CLBW}$$

$$\frac{\Omega(s)}{\Omega^*(s)} = \frac{\frac{K_P n_{\mathrm{pp}}\psi_{\mathrm{A}}}{J_s s^2} \frac{s+K_I}{\frac{s}{\mathrm{CLBW}}+1}}{1 + \frac{K_P n_{\mathrm{pp}}\psi_{\mathrm{A}}}{J_s s^2} \frac{s+K_I}{\frac{s}{\mathrm{CLBW}}+1}} = K_P \frac{n_{\mathrm{pp}}\psi_{\mathrm{A}}}{J_s} \frac{s+K_I}{\frac{1}{\mathrm{CLBW}} s^3 + s^2 + K_P \frac{n_{\mathrm{pp}}\psi_{\mathrm{A}}}{J_s}(s+K_I)}$$

Nested loop control system design in Freq. Domain

$$k_i = \frac{R}{L_q}$$
$$k_p = L_q \times \text{CLBW}$$
$$K_I = \frac{\text{CLBW}}{\delta^2}$$
$$K_P \frac{n_{\text{pp}} \psi_{\text{A}}}{J_s} = \delta K_I = \frac{\text{CLBW}}{\delta}$$

```
close all; cla; clc; s = zpk(0, [], 1);
% two tuning buttons
CLBW = 200; delta = 2;
% inner loop
Lq = 5e-3; R = 1; kp = CLBW * Lq; ki = R/Lq;
PCL = 1/(Lq * s+R); CCL = kp * (1+ki/s);
PclosedInner = CCL * PCL / (1+CCL * PCL)
% outer loop
n_pp = 4; psi_A = 0.1; Js = 0.006; K = n_pp* psi_A / Js;
KP = CLBW / delta / K; KI = CLBW/delta^2;
PVL = K/s; CVL = KP * (1+KI/s);
Popen = CVL * PclosedInner * PVL;
PopenMargin = allmargin(Popen)
P = Popen/(1+Popen)
P = minreal(P)
subplot(221); h = bodeplot(Popen); h.showCharacteristic('AllStal
subplot(222); bode(P); grid; h1 = findobj(gcf,'type','line'); set(h1,
subplot(223); step(P); grid; h1 = findobj(gcf,'type','line'); set(h1,'
subplot(224); pzmap(P); grid; h1 = findobj(gcf,'type','line'); set(h
```

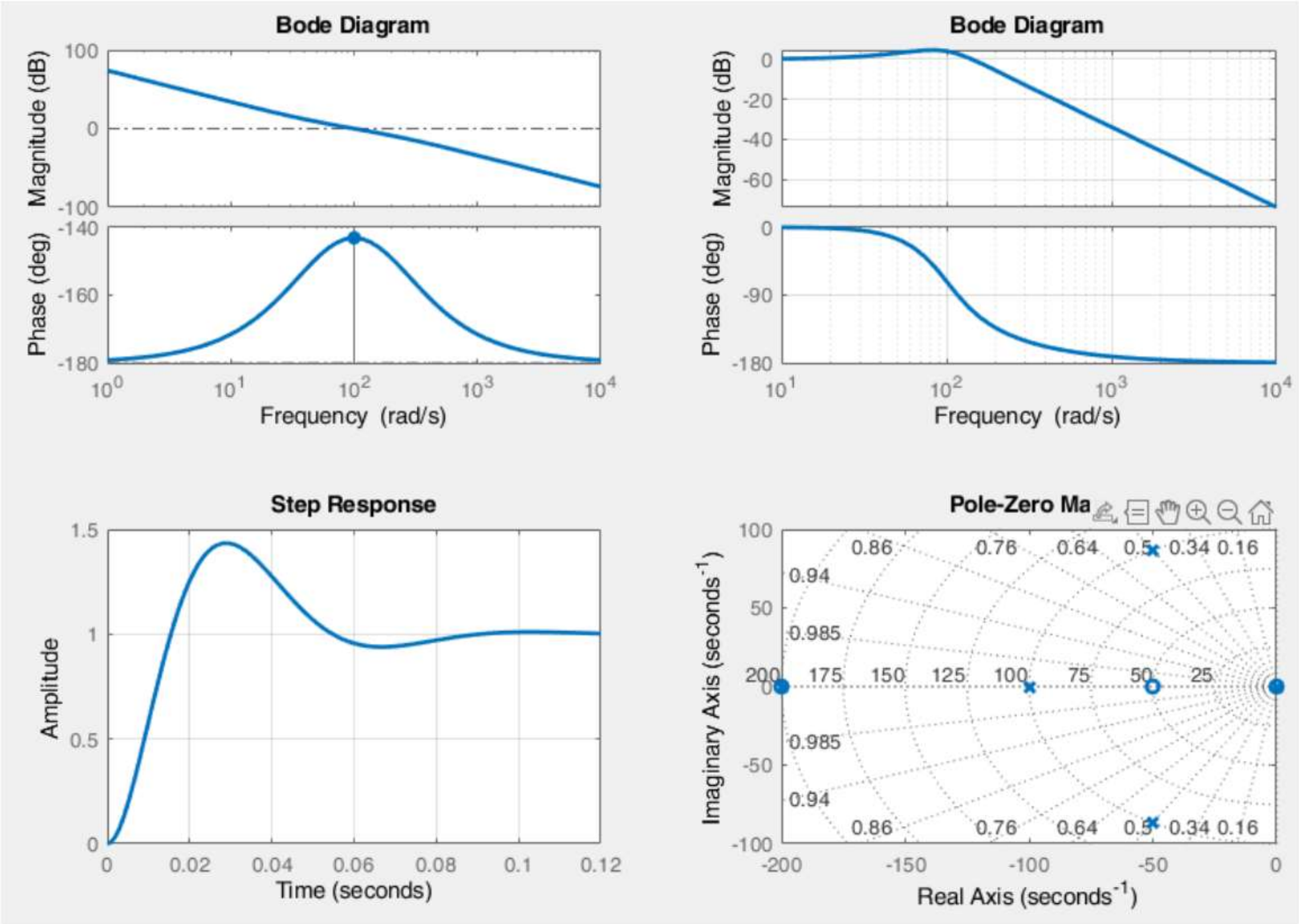


Figure 36. Nested loop control design with CLBW = 200 rad/s and $\delta = 2$. For motor parameters, see code snippets.

Nyquist Contour,
Nyquist Plot,
and Nyquist Criterion

(not a necessary tool)

Conformal Mapping (Angle-Preserving Mapping)

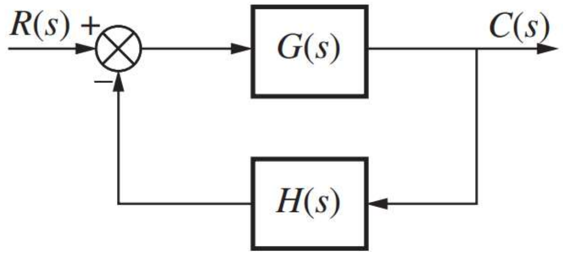


FIGURE 10.20 Closed-loop control system

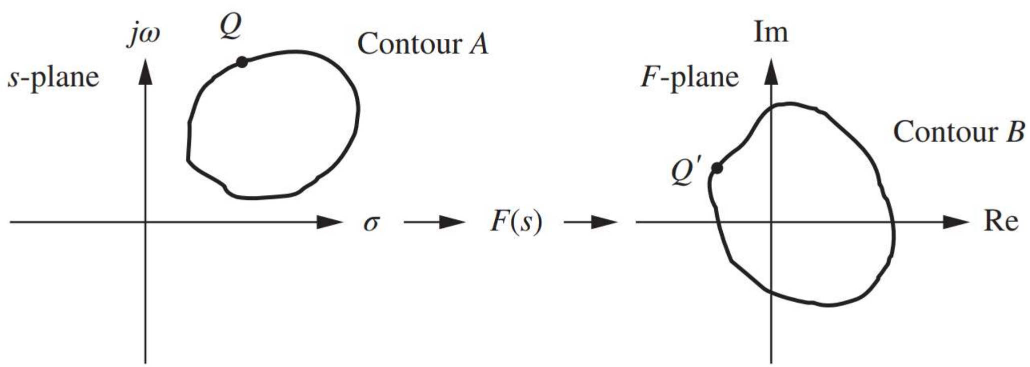


FIGURE 10.21 Mapping contour A through function $F(s)$ to contour B

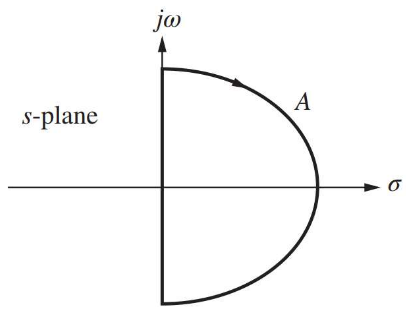


FIGURE 10.24 Contour enclosing right half-plane to determine stability

Stability Margins in Nyquist Plot

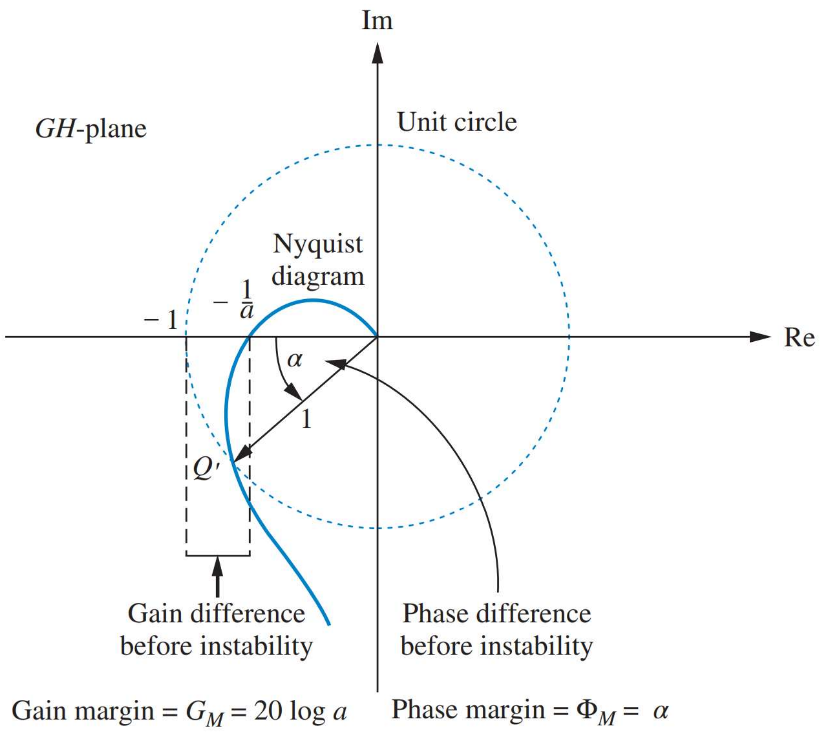


FIGURE 10.35 Nyquist diagram showing gain and phase margins