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Lecture 4: Compensator/Controller

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SIST 1D#206

Compensator/Controller

(adding pole and zero to system
for improvement of steady state error
and transient performance)

Dynamic Compensator

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- In standard control problem considered for root locus, the controller is a pure scalar gain. Such a controller is also called a static compensator or proportional regulation/control.
- The word dynamic describes compensators with noninstantaneous transient response. The transfer functions of such compensators are functions of the Laplace variable, s, rather than pure gain.
  - For example, the parallel version of the PI controller and PD controller are:

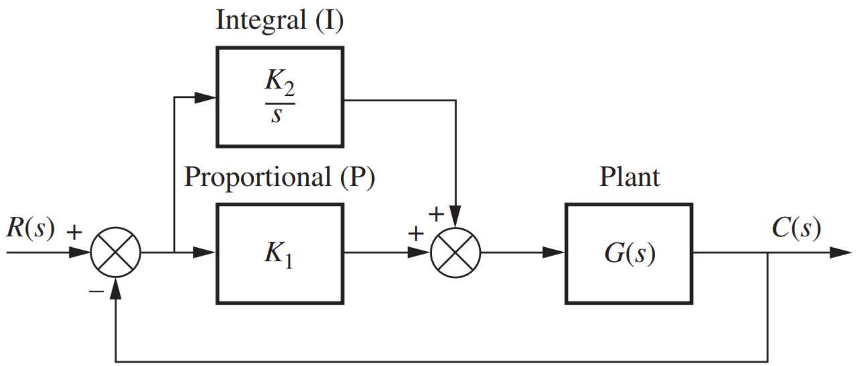

FIGURE 9.8 PI controller

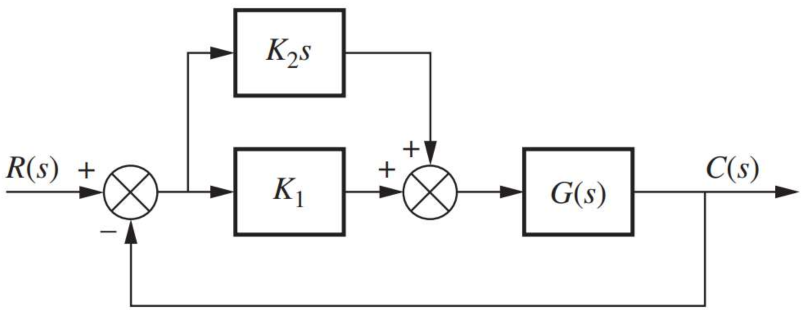

FIGURE 9.23 PD controller

Integral and Lag Compensation

Ideal Integral Compensation (PI)

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- A compensator with a pole at the origin and a zero close to the pole is called an **ideal integral compensator**.

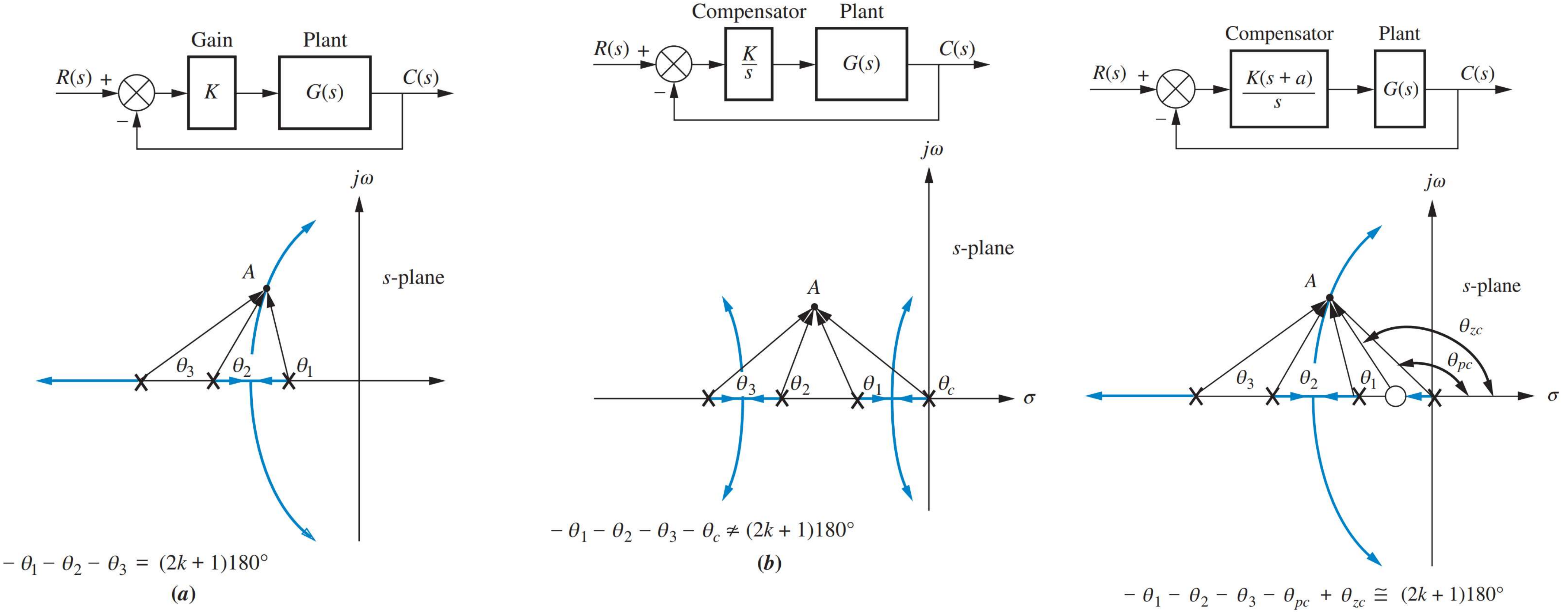

FIGURE 9.3 Pole at A is **a.** on the root locus without compensator; **b.** not on the root locus with compensator pole added; **c.** approximately on the root locus with compensator pole and zero added

Ideal Integral Compensation (PI)

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- PI compensator is possible to eliminate steady state error

PROBLEM: Given the system of Figure 9.4(*a*), operating with a damping ratio of 0.174, show that the addition of the ideal integral compensator shown in Figure 9.4(*b*) reduces the steady-state error to zero for a step input without appreciably affecting transient response. The compensating network is chosen with a pole at the origin to increase the system type and a zero at −0.1, close to the compensator pole, so that the angular contribution of the compensator evaluated at the original, dominant, second-order poles is approximately zero. Thus, the original, dominant, second-order closed-loop poles are still approximately on the new root locus.

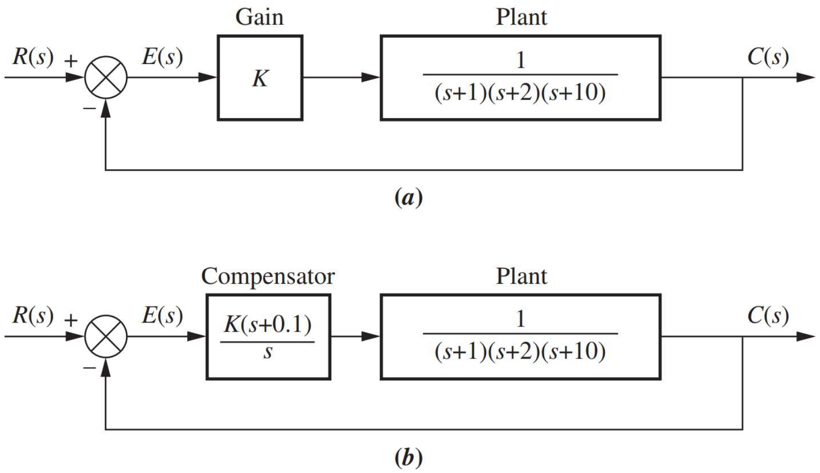

FIGURE 9.4 Closed-loop system for Example 9.1: **a.** before compensation; **b.** after ideal integral compensation

SOLUTION: We first analyze the uncompensated system and determine the location of the dominant, second-order poles. Next we evaluate the uncompensated steady-state error for a unit step input. The root locus for the uncompensated system is shown in Figure 9.5.

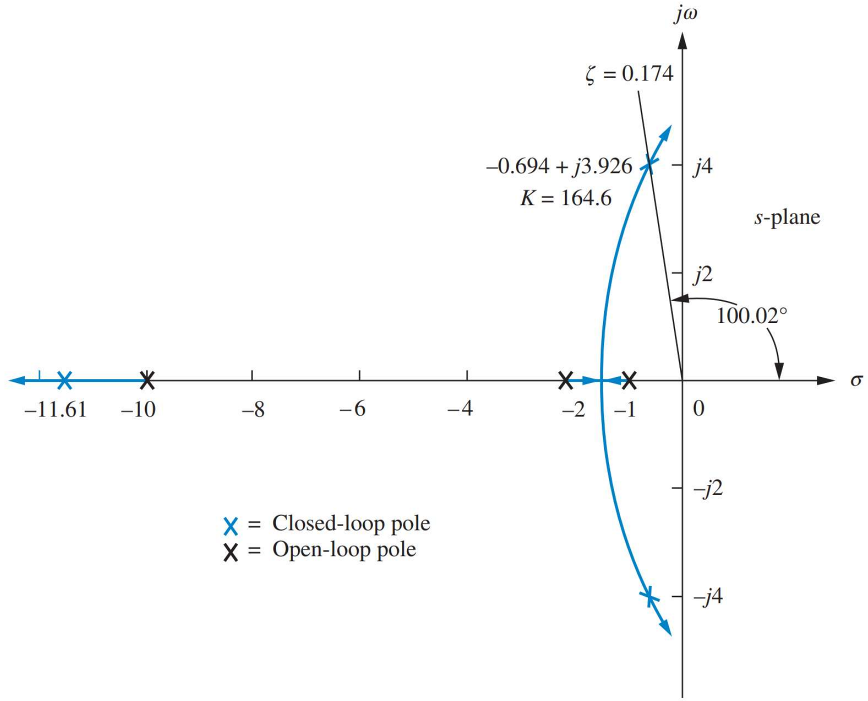

FIGURE 9.5 Root locus for uncompensated system of Figure 9.4(*a*)

Ideal Integral Compensation (PI)

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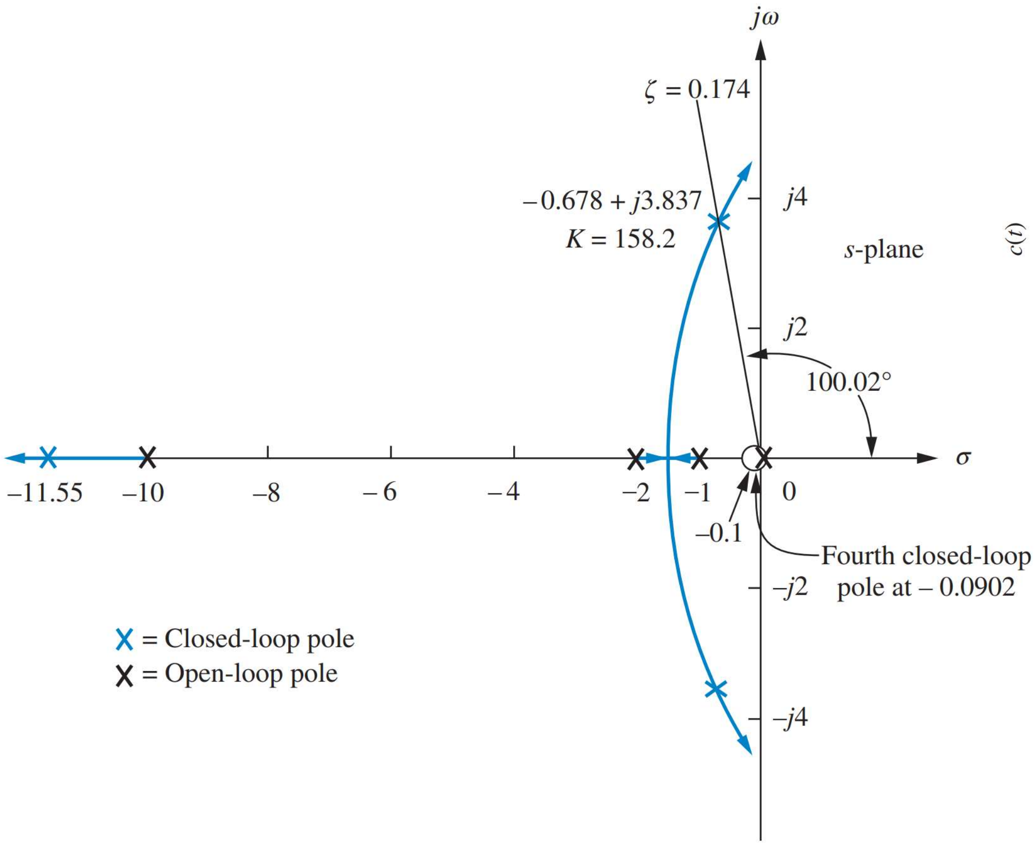
- PI compensator is possible to eliminate steady state error



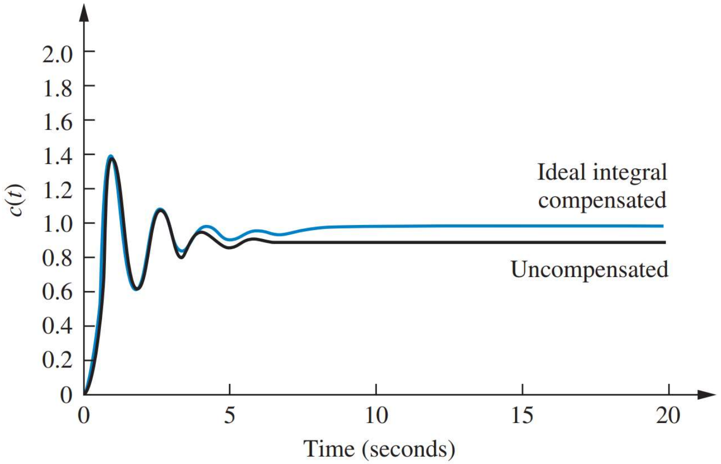


FIGURE 9.7 Ideal integral compensated system response and the uncompensated system response of Example 9.1

FIGURE 9.6 Root locus for compensated system of Figure 9.4(*b*)

Lag Compensation

$$\theta = \sum \text{zero angles} - \sum \text{pole angles} = \sum_{i=1}^{m} \angle(s+z_i) - \sum_{j=1}^{n} \angle(s+p_j)$$

- Lag compensation is similar to the PI compensation but has a nonzero compensator pole to the right of the compensator zero, and **it adds net lag to the transfer function phase.**
- When the pole-zero pair is close to the origin, we have:
  - Transient response does not change a lot.
  - The value of K does not change a lot.
  - Steady state error constant is multiplied by a factor of z_c/p_c, i.e., $K_{v_N} = K_{v_O}\dfrac{z_c}{p_c} > K_{v_O}$

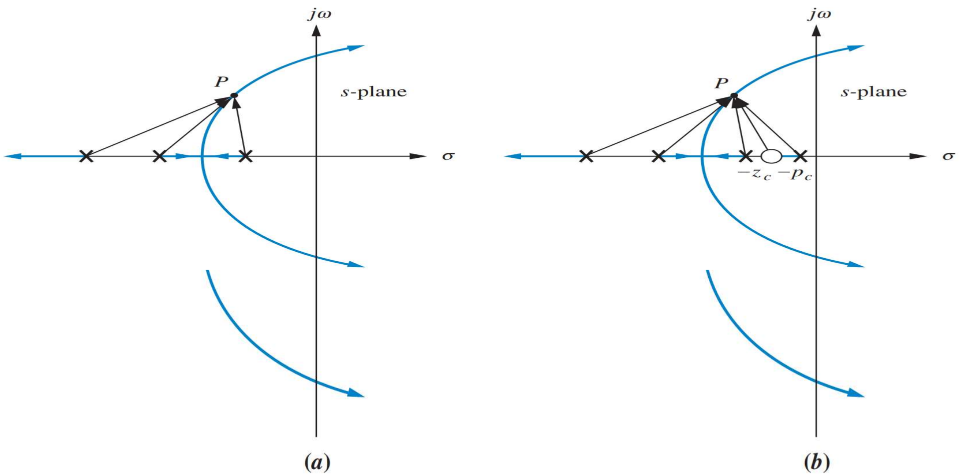
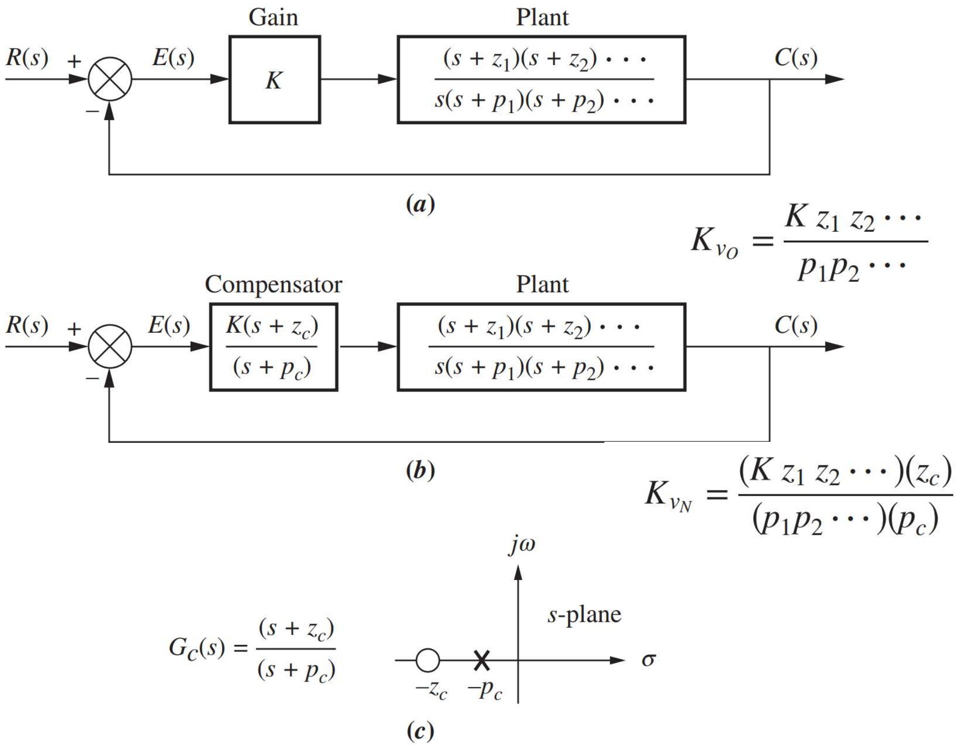


FIGURE 9.10 Root locus: **a.** before lag compensation; **b.** after lag compensation



$$K_{v_O} = \frac{K z_1 z_2 \cdots}{p_1 p_2 \cdots}$$

$$K_{v_N} = \frac{(K z_1 z_2 \cdots)(z_c)}{(p_1 p_2 \cdots)(p_c)}$$

FIGURE 9.9 **a.** Type 1 uncompensated system; **b.** Type 1 compensated system; **c.** compensator pole-zero plot

Lag Compensation

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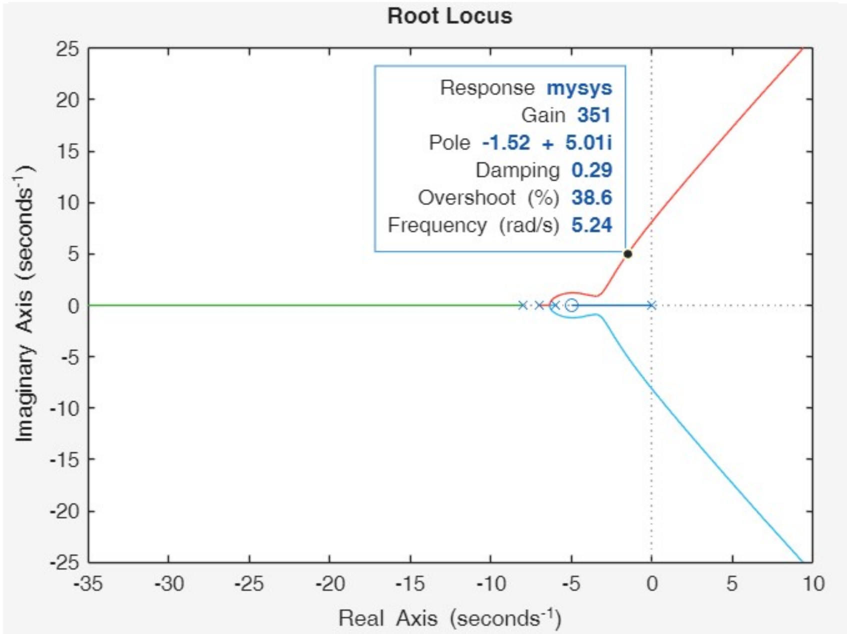
- Let's revisit the problem of a very large error constant specification.
- Can you use lag compensation to meet the error constant specification?
- Use Matlab to assist:

```
>> G = (s+5)/s/(s+6)/(s+7)/(s+8)
>> rlocus(G)
```



Gain Design to Meet a Steady-State Error Specification

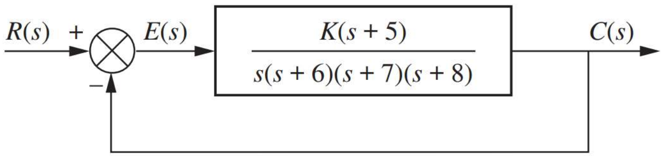


FIGURE 7.10 Feedback control system for Example 7.6

PROBLEM: Given the control system in Figure 7.10, find the value of K so that there is 0.1% error in the steady state.

SOLUTION: Since the system is Type 1, the error stated in the problem must apply to a ramp input; only a ramp yields a finite error in a Type 1 system. Thus,

$$e(\infty) = \frac{1}{K_v} = 0.1\% \qquad (7.55)$$

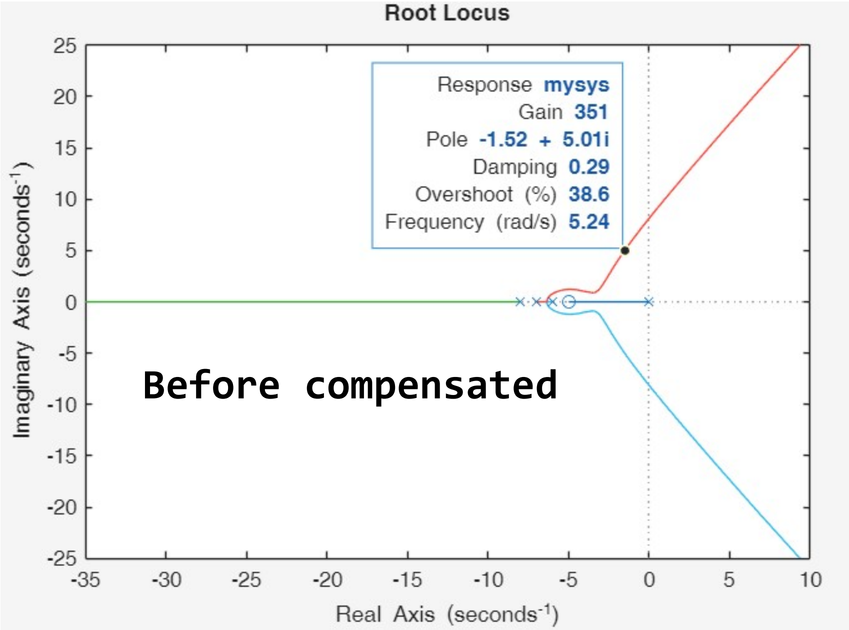
Lag Compensation

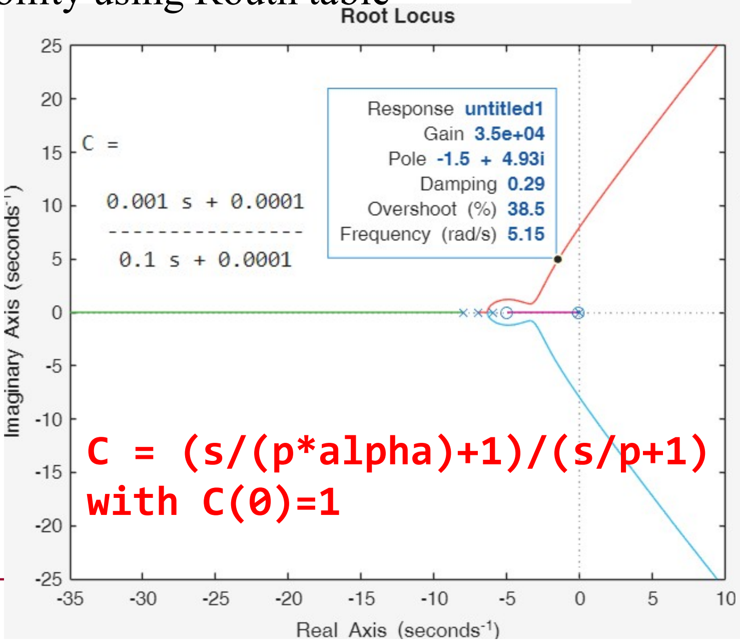
- Candidate lag compensator is

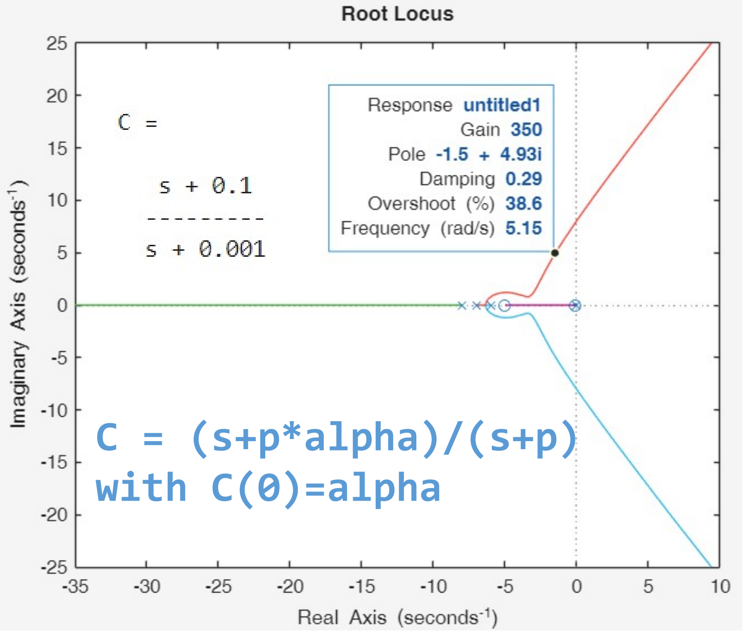
```
>> s=tf('s'); G = (s+5)/s/(s+6)/(s+7)/(s+8)
>> alpha=100; p=0.001; C = (s+p*alpha)/(s+p)
>> alpha=100; p=0.001; C = (s/(p*alpha)+1)/(s/p+1)
>> rlocus(C*G)
```

- Velocity error constant is set to 1000.
  - Error constant is calculated to be K * 5 / (6*7*8) * alpha
  - We have $K = 672$ and check stability using Routh table

$$K_{v_N} = \frac{(K\, z_1\, z_2 \cdots)(z_c)}{(p_1 p_2 \cdots)(p_c)}$$







Lag Compensation

PROBLEM: A unity feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 15% overshoot. Do the following:

a. Evaluate the steady-state error for a unit ramp input.

b. Design a lag compensator to improve the steady-state error by a factor of 20.

c. Evaluate the steady-state error for a unit ramp input to your compensated system.

d. Evaluate how much improvement in steady-state error was realized.

ANSWERS:

a.

b.

c.

d.

Differentiation and Lead Compensation

Ideal Derivative Compensation (PD)

- A compensator of a **zero** is called an **ideal derivative compensator**.

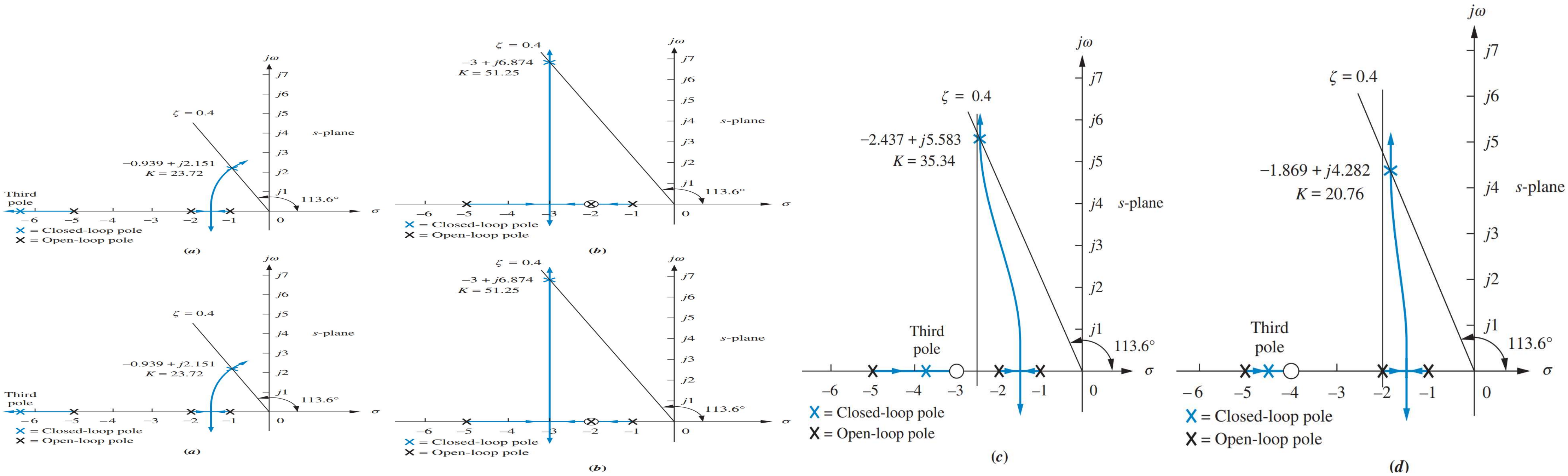


FIGURE 9.15 Using ideal derivative compensation: **a.** uncompensated; **b.** compensator zero at −2; **c.** compensator zero at −3; **d.** compensator zero at −4

Ideal Derivative Compensation (PD)

- A compensator of a **zero** is called an **ideal derivative compensator**.

TABLE 9.2 Predicted characteristics for the systems of Figure 9.15

	Uncompensated	Compensation b	Compensation c	Compensation d
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom, poles				
K				
ζ				
ω_n				
$\%OS$				
T_s				
T_p				
K_p				
$e(\infty)$				
Third pole				
Zero				
Comments				

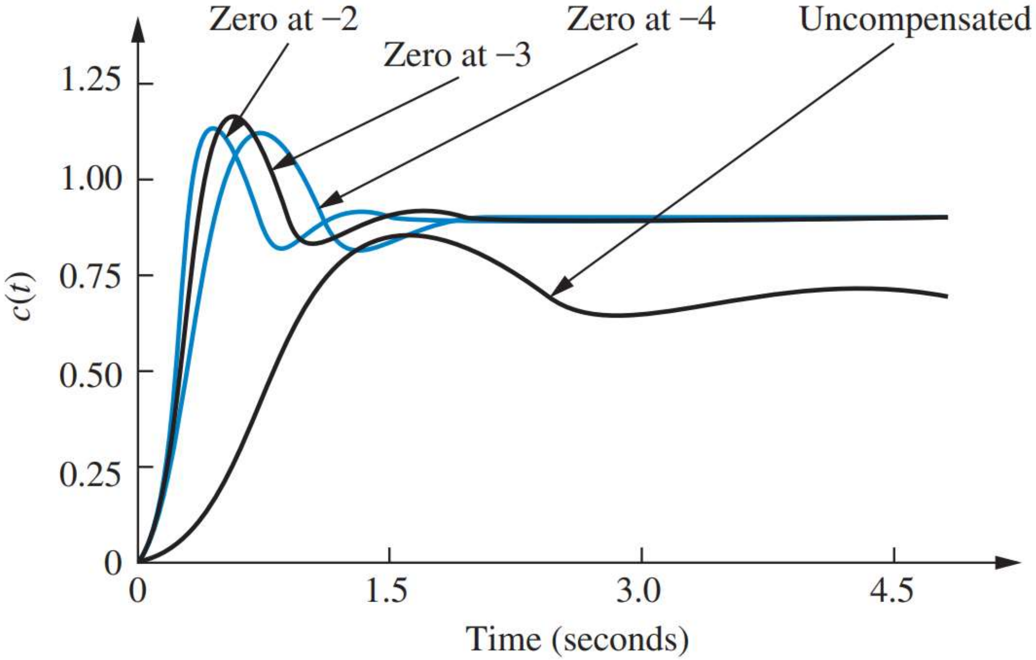


FIGURE 9.16 Uncompensated system and ideal derivative compensation solutions from Table 9.2

Ideal Derivative Compensation (PD)

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PROBLEM: Given the system of Figure 9.17, design an ideal derivative compensator to yield a 16% overshoot, with a threefold reduction in settling time.

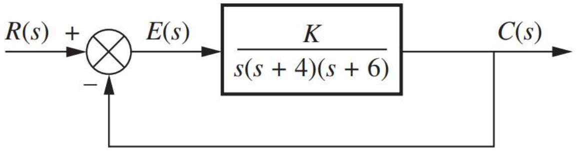

FIGURE 9.17 Feedback control system for Example 9.3

SOLUTION: Let us first evaluate the performance of the uncompensated system operating with 16% overshoot. The root locus for the uncompensated system is shown in Figure 9.18. Since 16% overshoot is equivalent to $\zeta = 0.504$, we search along that damping ratio line for an odd multiple of 180° and find that the dominant, second-order pair of poles is at $-1.205 \pm j2.064$. Thus, the settling time of the uncompensated system is

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320 \qquad \omega_n = \sqrt{1.204^2 + 2.064^2} = 2.3895$$

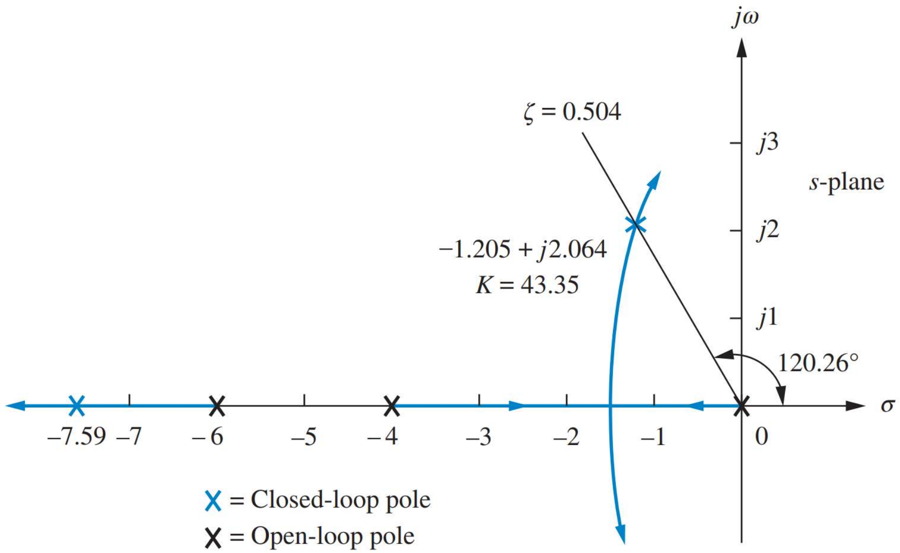

FIGURE 9.18 Root locus for uncompensated system shown in Figure 9.17

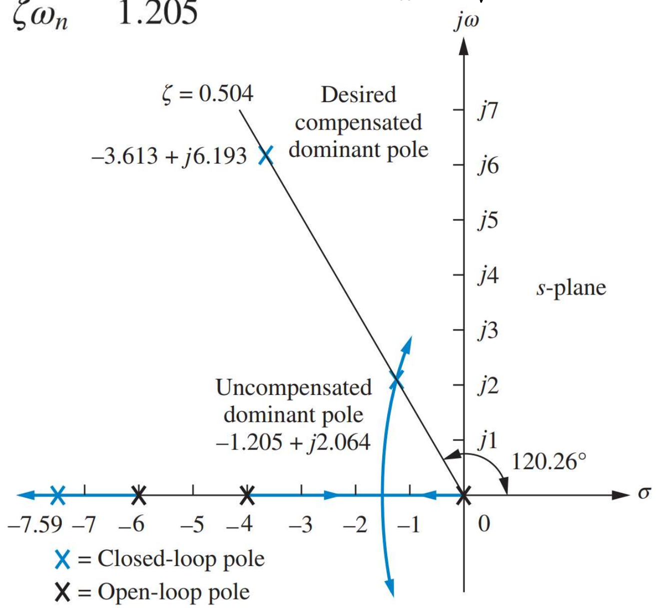

FIGURE 9.19 Compensated dominant pole superimposed over the uncompensated root locus for Example 9.3

Ideal Derivative Compensation (PD)

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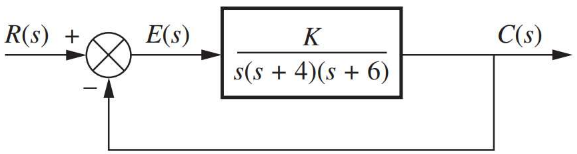


FIGURE 9.17 Feedback control system for Example 9.3

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$$

$$\omega_n = \sqrt{1.204^2 + 2.064^2} = 2.3895$$

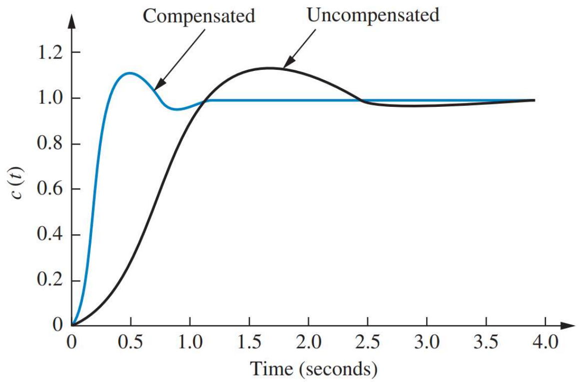


FIGURE 9.22 Uncompensated and compensated system step responses of Example 9.3

$$\angle L(\square) = \angle(\square + z_c) - \angle\,\square(\square + 4)\,(\square + 6) = 180^\circ$$

$$\Rightarrow \angle(\square + z_c) = 180^\circ + \angle\,\square(\square + 4)\,(\square + 6) = 95.6^\circ$$

$$\frac{6.193}{3.613 - \sigma} = \tan(180^\circ - 95.6^\circ)$$

$$\sigma = 3.006.$$

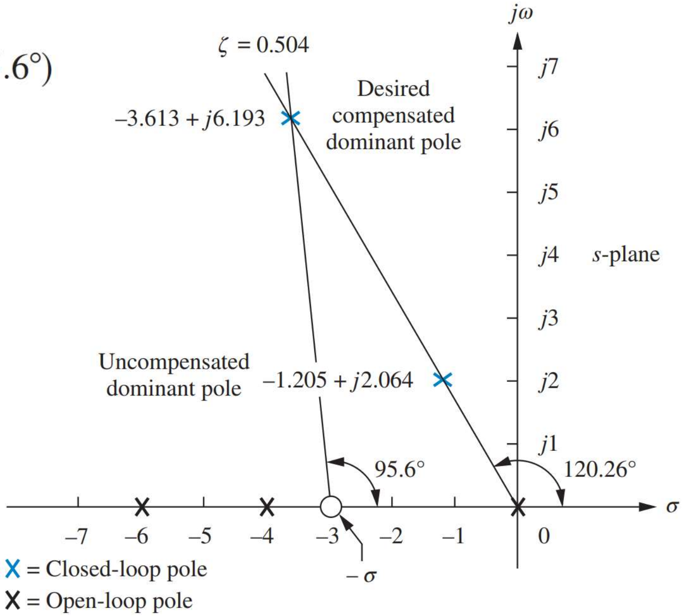


FIGURE 9.20 Evaluating the location of the compensating zero for Example 9.3

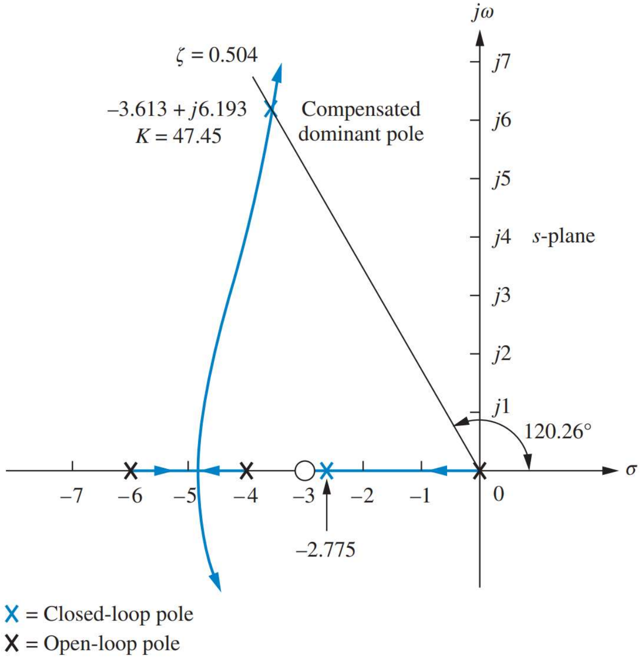


FIGURE 9.21 Root locus for the compensated system of Example 9.3

Ideal Derivative Compensation (PD)

- Here is the standard procedure I prefer:
  - *Note the PD compensation is able to provide a lead angle of 95.6 degrees, which is a lot.*

猜测ω_n，也就是主导极点距离原点的距离，

未动态校正的根这样算：$s=\omega_n\left(\zeta+j\sqrt{1-\zeta^2}\right)$

特征方程：$1+G(s)=0$

$$\Rightarrow G(s)=\frac{K}{s(s+4)(s+6)}=-1$$

验证s是否符合角度条件：

$$\Rightarrow \angle G(s)=-\angle s(s+4)(s+6)=180^\circ$$

确定s的值，然后计算希望动态校正到哪里：

$\square=$横坐标$+j$纵坐标

- 横坐标基于调节时间T_s给定，然后根据ζ斜线求纵坐标
- 纵坐标基于峰值时间T_p给定，然后根据ζ斜线求恒坐标

特征方程：$1+L(s)=0$

$$\Rightarrow L(s)=C(s)G(s)=(s+z_c)\frac{K}{s(s+4)(s+6)}=-1$$

根据角度条件求PD控制器的零点放哪里：

$$\Rightarrow \angle L(\square)=\angle(\square+z_c)-\angle\square(\square+4)(\square+6)=180^\circ$$

$$\Rightarrow \angle(\square+z_c)=180^\circ+\angle\square(\square+4)(\square+6)=95.6^\circ$$

Lead Compensation

- An active ideal derivative compensator can be approximated with a passive lead compensator (i.e., adding a zero and a pole)
  - if the pole is farther from the imaginary axis than the zero, the angular contribution of the compensator is still positive and thus approximates an equivalent single zero.
  - The disadvantage is that the additional pole does not reduce the number of branches of the root locus that cross the imaginary axis into the right half-plane.
  - Different lead compensators correspond to different static error constants and K values.

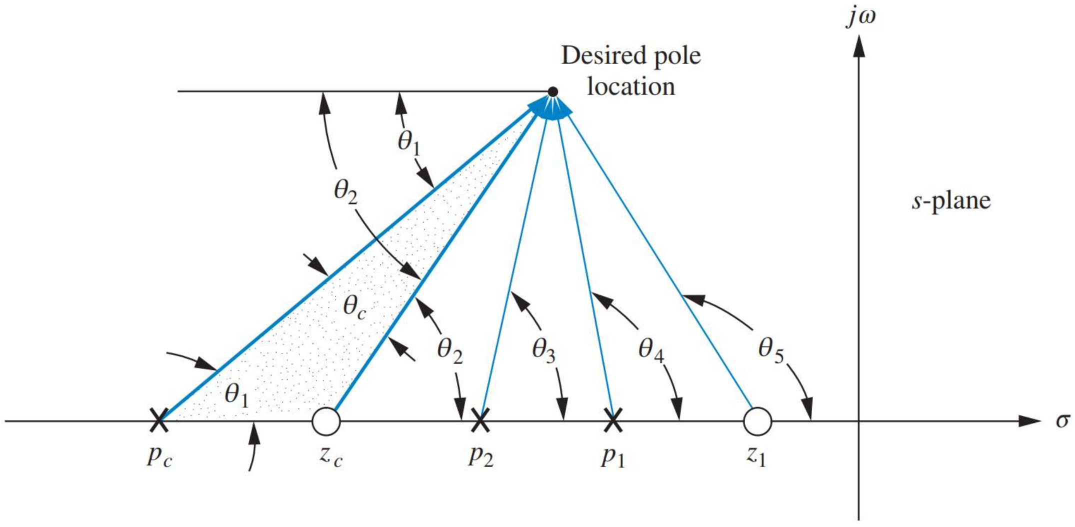


FIGURE 9.24 Geometry of lead compensation

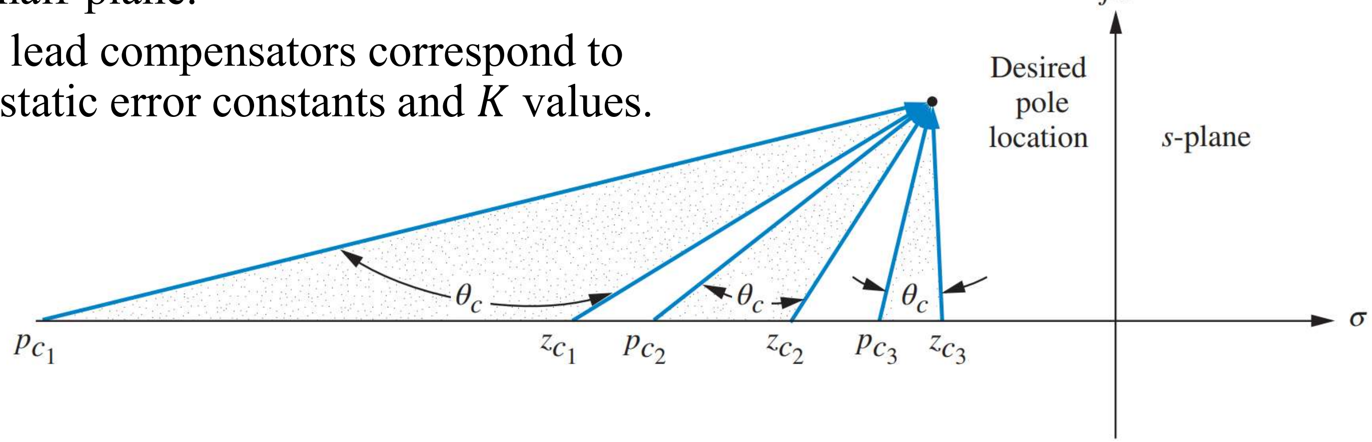


FIGURE 9.25 Three of the infinite possible lead compensator solutions

Lead Compensation

Lead Compensator Design

PROBLEM: Design a lead compensator for the system of Figure 9.17 that will reduce the settling time by a factor of 2 while maintaining 30% overshoot. Compare the system characteristics between the three designs.

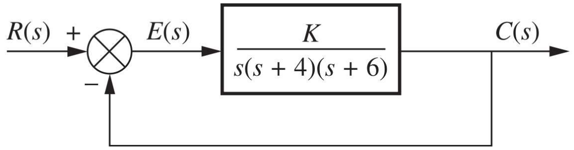


FIGURE 9.17 Feedback control system for

SOLUTION: First determine the characteristics of the uncompensated system operating at 30% overshoot to see what the uncompensated settling time is. Since 30% overshoot is equivalent to a damping ratio of 0.358, we search along the $\zeta = 0.358$ line for the uncompensated dominant poles on the root locus, as shown in Figure 9.26. From the pole's real part, we calculate the uncompensated settling time as $T_s = 4/1.007 = 3.972$ seconds. The remaining characteristics of the uncompensated system are summarized in Table 9.4.

Next we find the design point. A twofold reduction in settling time yields $T_s = 3.972/2 = 1.986$ seconds, from which the real part of the desired pole location is $-\zeta\omega_n = -4/T_s = -2.014$. The imaginary part is $\omega_d = -2.014\tan(110.98°) = 5.252$.

We continue by designing the lead compensator. Arbitrarily assume a compensator zero at −5 on the real axis as a possible solution. Using the root locus program, sum the angles from both this zero and the uncompensated system's poles and zeros, using the design point as a test point. The resulting angle is −172.69°. The difference between this angle and 180° is the angular contribution required from the compensator pole in order to place the design point on the root locus. Hence, an angular contribution of −7.31° is required from the compensator pole.

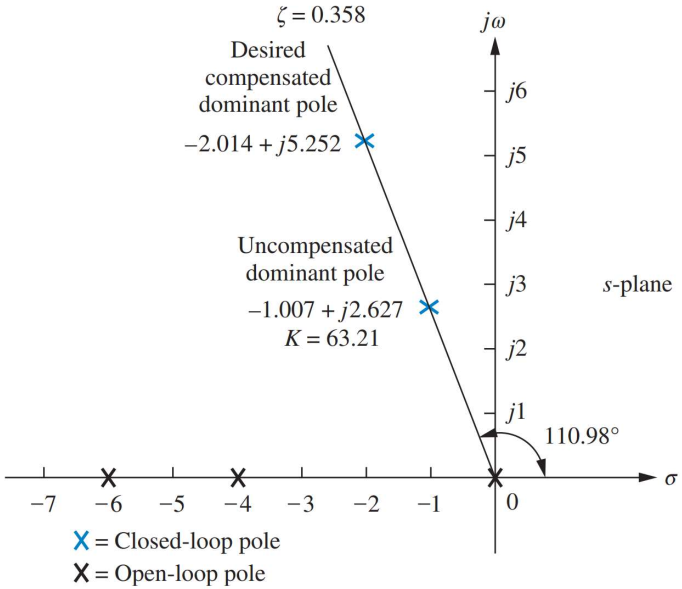
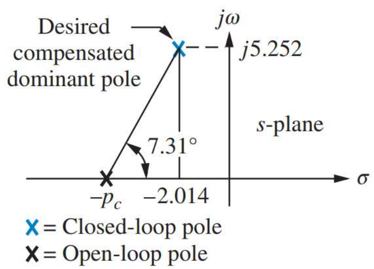


FIGURE 9.26 Lead compensator design, showing evaluation of uncompensated and compensated dominant poles for Example 9.4

TABLE 9.4 Comparison of lead compensation designs for Example 9.4

	Uncompensated	Compensation a
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$
K	63.21	1423
ζ	0.358	0.358
ω_n	2.813	5.625
$\%OS^*$	30 (28)	30 (30.7)
T_s^*	3.972 (4)	1.986 (2)
T_p^*	1.196 (1.3)	0.598 (0.6)
K_v	2.634	6.9
$e(\infty)$	0.380	0.145
Other poles	−7.986	−43.8, −5.134
Zero	None	−5
Comments	Second-order approx. OK	Second-order approx. OK

*Simulation results are shown in parentheses.



Note: This figure is not drawn to scale.

$$\frac{5.252}{p_c - 2.014} = \tan 7.31°$$

$$p_c = 42.96$$

FIGURE 9.27 s-plane picture used to calculate the location of the compensator pole for Example 9.4

Lead Compensation

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- Here is the standard procedure I prefer:
  - *Note as compared with PD compensation, the lead compensation is able to provide a limited lead angle.*

猜测ω_n，也就是主导极点距离原点的距离，

未动态校正的根这样算：$s=\omega_n\left(\zeta+j\sqrt{1-\zeta^2}\right)$

特征方程：$1+G(s)=0$

$$\Rightarrow G(s)=\frac{K}{s(s+4)(s+6)}=-1$$

验证s是否符合角度条件：

$$\Rightarrow \angle G(s)=-\angle s(s+4)(s+6)=180^\circ$$

确定s的值，然后计算希望动态校正到哪里：

$\square=$横坐标$+j$纵坐标

- 横坐标基于调节时间T_s给定，然后根据ζ斜线求纵坐标
- 纵坐标基于峰值时间T_p给定，然后根据ζ斜线求恒坐标

特征方程：$1+L(s)=0$

$$\Rightarrow L(s)=C(s)G(s)=\frac{(s+z_c)}{(s+p_c)}\frac{K}{s(s+4)(s+6)}=-1$$

根据角度条件求PD控制器的零点放哪里：

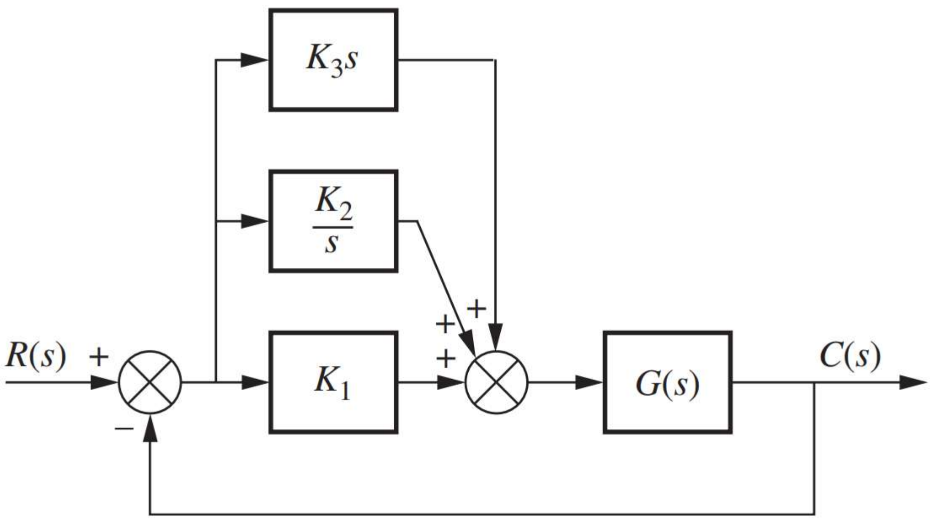
$$\Rightarrow \angle L(\square)=\angle(\square+z_c)-(\square+p_c)-\angle\square(\square+4)(\square+6)=180^\circ$$

$$\Rightarrow \angle(\square+z_c)-(\square+p_c)=180^\circ+\angle\square(\square+4)(\square+6)$$

PID Compensation

PID Control (parallel version)

- This textbook first designs PD for transient performance improvement and then designs PI for steady state performance improvement,
  - *which is called proportional-plus-integral-plus-derivative* (PID) *controller*.
  - The parallel version of the PID regulator is shown in FIGURE 9.30.



$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s} = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3}\right)}{s}$$

FIGURE 9.30 PID controller

PID Control (parallel version)

PROBLEM: Given the system of Figure 9.31, design a PID controller so that the system can operate with a peak time that is two-thirds that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input.

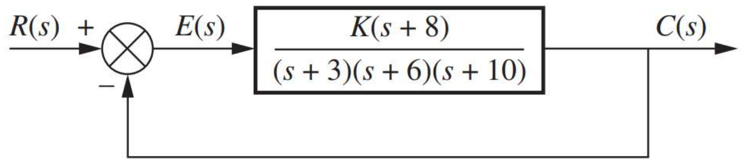


FIGURE 9.31 Uncompensated feedback control system for Example 9.5

- The desired dominant poles are located at

$$\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$$

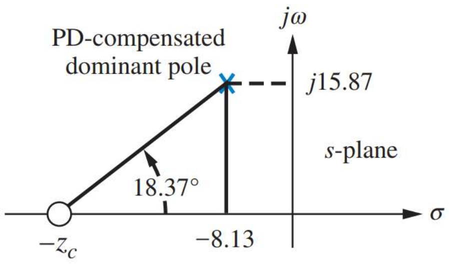
$$\sigma = \frac{\omega_d}{\tan 117.13°} = -8.13$$

- Find the sum of angles from **the uncompensated system's poles and zeros** to **the desired compensated dominant pole** to be 198.37°, which suggests the PD compensator adds angular contribution of 198.37 – 180 degrees.

$$\frac{15.87}{z_c - 8.13} = \tan 18.37°$$

$$z_c = 55.92$$

$$G_{PD}(s) = (s + 55.92)$$



Note: This figure is not drawn to scale.

FIGURE 9.33 Calculating the PD compensator zero for Example 9.5

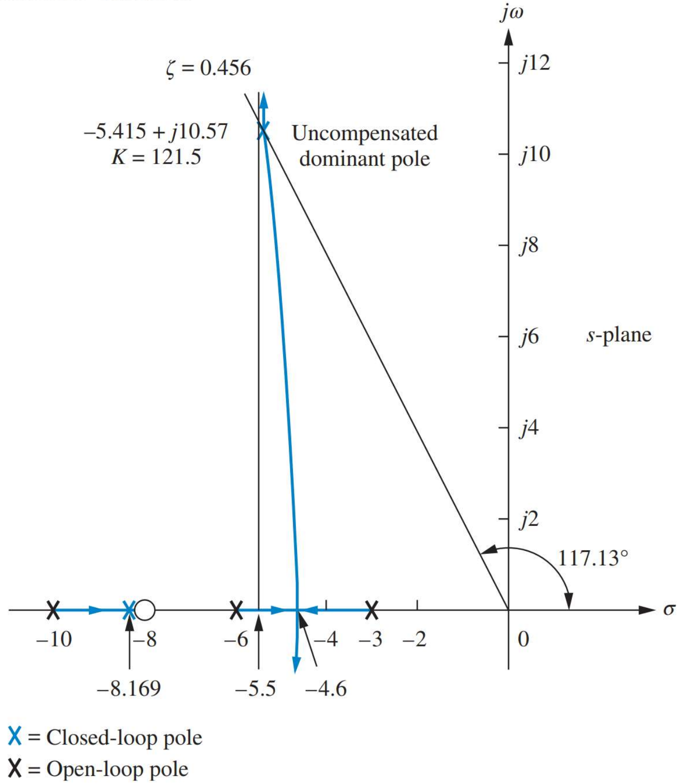


FIGURE 9.32 Root locus for the uncompensated system of Example 9.5

PID Control (parallel version)

- PD adds a pure zero that attracts the root locus to the left the s-plane:

$$G_{PD}(s) = (s + 55.92)$$

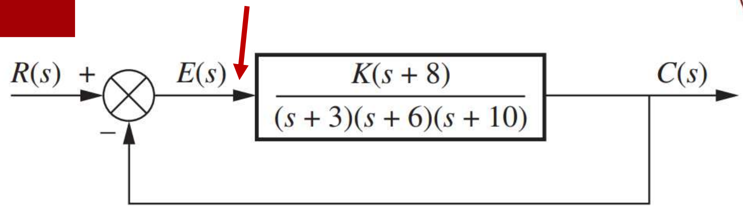
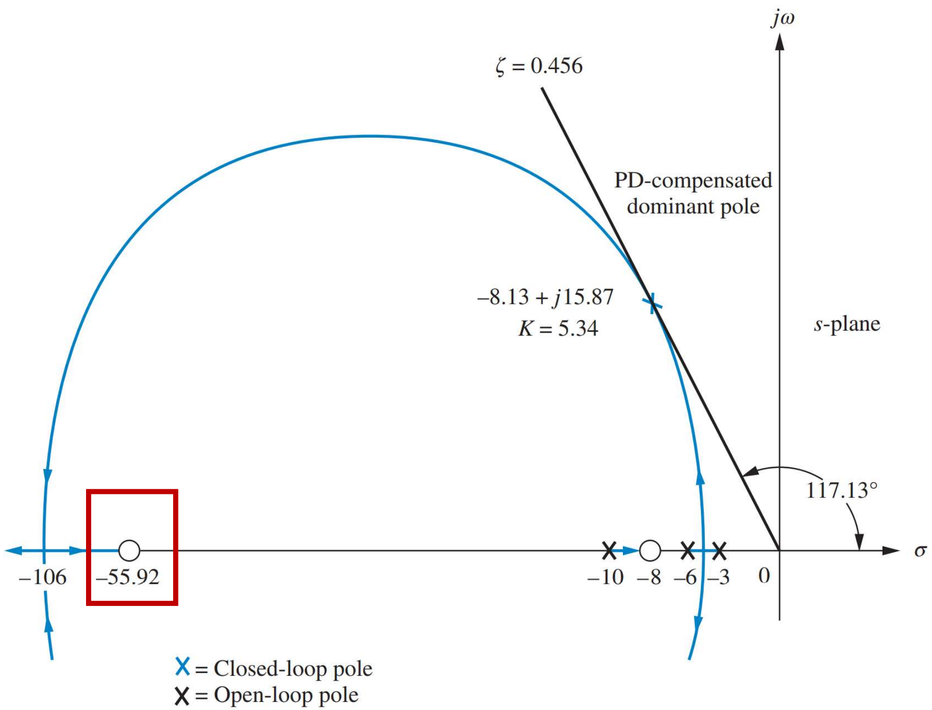


FIGURE 9.31 Uncompensated feedback control system for Example 9.5



Note: This figure is not drawn to scale.

FIGURE 9.34 Root locus for PD-compensated system of Example 9.5

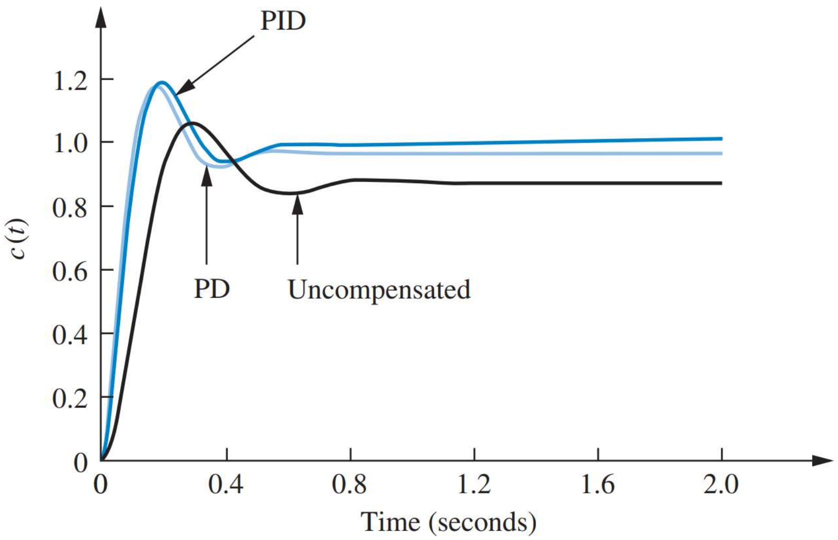


FIGURE 9.35 Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5

PID Control (parallel version)

$G_{PD}(s) = (s + 55.92)$ $G_{PI}(s) = \dfrac{s + 0.5}{s}$

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- PI adds an integrator (pole at origin) and a zero, which corresponds to the **slow transient response during the settling phase.**

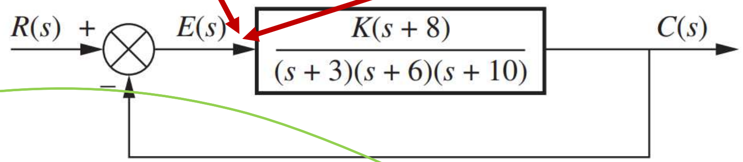

FIGURE 9.31 Uncompensated feedback control system for Example 9.5

$$G_{PID}(s) = \frac{K(s+55.92)(s+0.5)}{s} = \frac{4.6(s+55.92)(s+0.5)}{s} = \frac{4.6(s^2+56.42s+27.96)}{s}$$

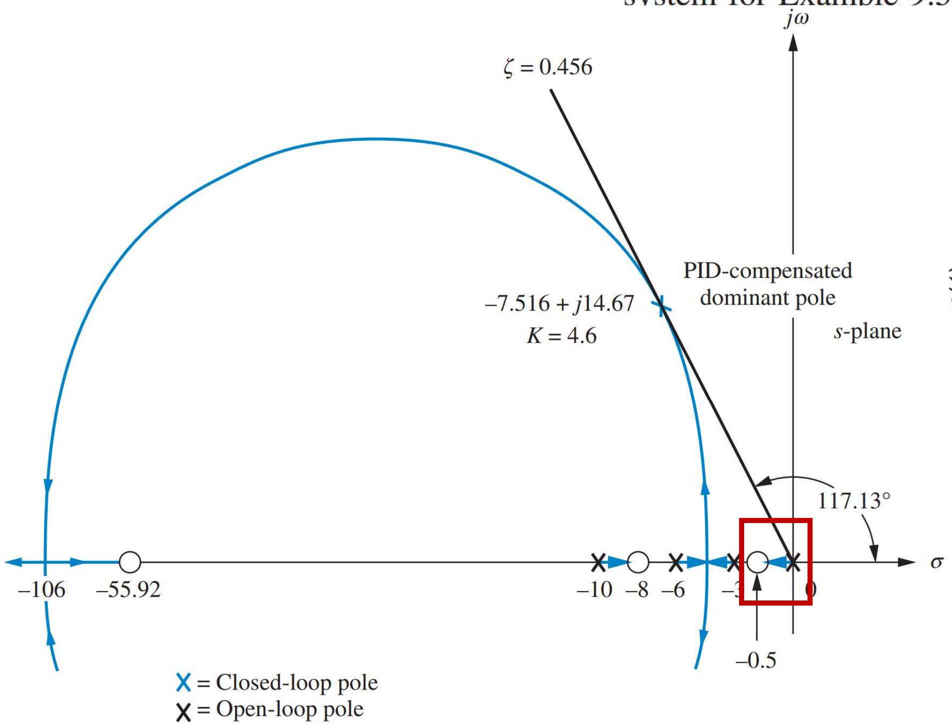

FIGURE 9.36 Root locus for PID-compensated system of Example 9.5

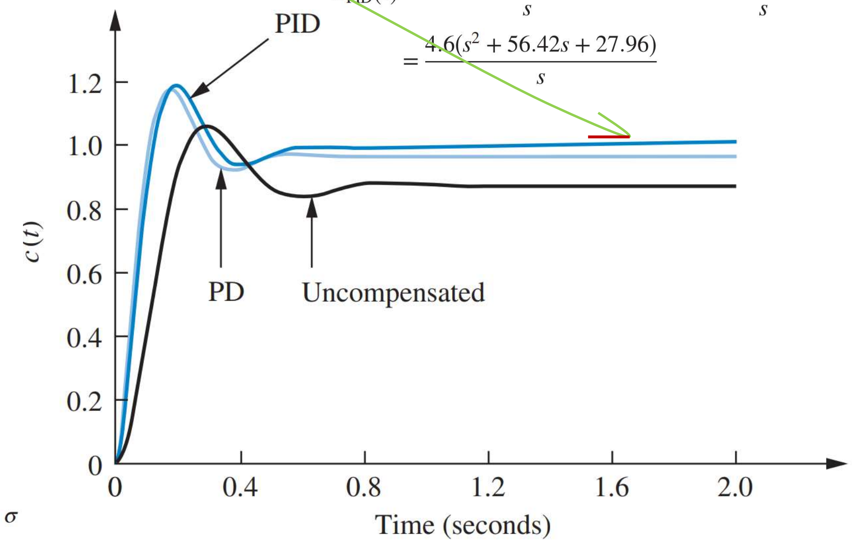

FIGURE 9.35 Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5

Lag-Lead Compensation

Lag-Lead Compensation

- If we design a **passive lead compensator** and then design **a passive lag compensator**, the resulting compensator is called a lag-lead compensator.
  - Let's go through the process via an example.

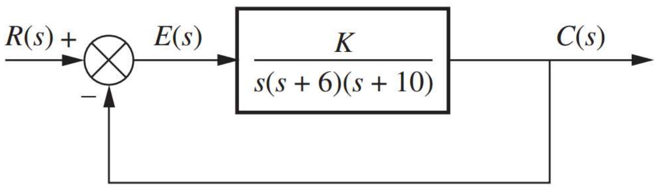

FIGURE 9.37 Uncompensated system for Example 9.6

PROBLEM: Design a lag-lead compensator for the system of Figure 9.37 so that the system will operate with 20% overshoot and a twofold reduction in settling time. Further, the compensated system will exhibit a tenfold improvement in steady-state error for a ramp input.

One idea is to use the compensator zero to cancel out a system pole. In other words, lead compensator equivalently moves the open loop pole to the left of the s-plane, which adds net angular contribution to the transfer function.

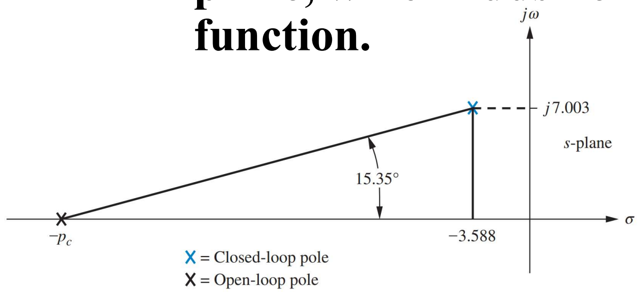

FIGURE 9.39 Evaluating the compensator pole for Example 9.6

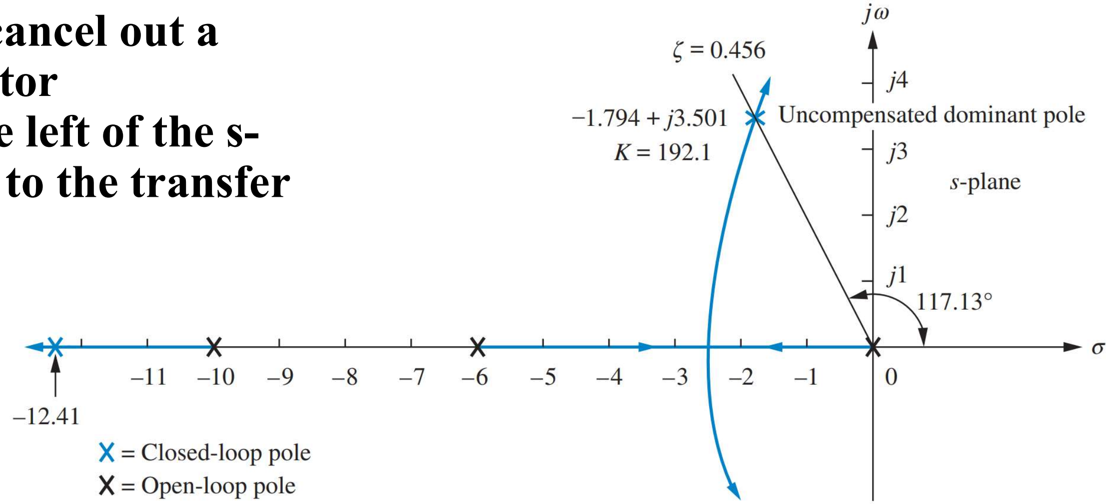

FIGURE 9.38 Root locus for uncompensated system of Example 9.6

Lag-Lead Compensation

- One idea is to use the compensator zero to cancel out a system pole. In other words, lead compensator equivalently moves the open loop pole to the left of the s-plane, which adds net angular contribution to the transfer function, as indicated by the curly arrow in FIGURE 9.40.
  - Sum the angles to the design point from the uncompensated system's poles and zeros and the compensator zero and get $-164.65°$. The difference between 180° and this quantity is the angular contribution required from the compensator pole, i.e., $-15.35°$.
  - The lead compensated transfer function is:

$$G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)}$$

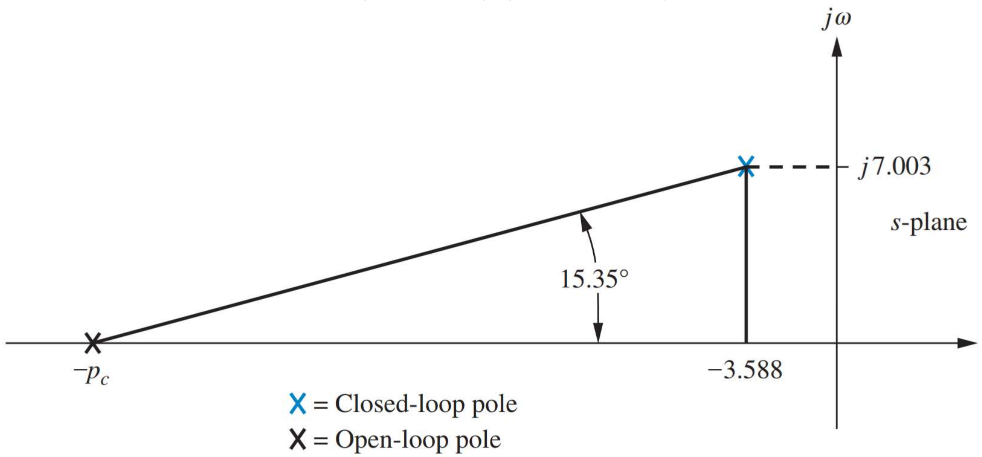

FIGURE 9.39 Evaluating the compensator pole for Example 9.6

$$\frac{7.003}{p_c - 3.588} = \tan 15.35°$$

p_c, is found to be -29.1.

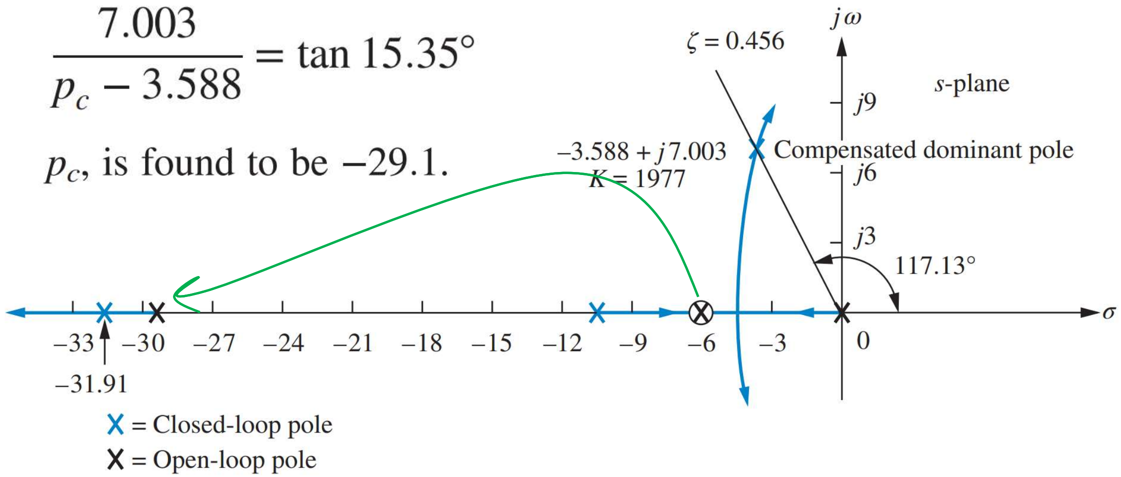

FIGURE 9.40 Root locus for lead-compensated system of Example 9.6

Lag-Lead Compensation

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- The velocity error constant of $G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)}$ is 1977 / 10/29.1 = 6.793
- The velocity error constant of the uncompensated $G(s) = \frac{192.1}{s(s+6)(s+10)}$ is $\frac{192.1}{6\times 10} = 3.201$
- Now to improve velocity error constant up to tenfold, we first **arbitrarily** choose the **lag compensator** pole at 0.01 and then places the **lag compensator** zero at 0.04713, i.e., $G_{lag}(s) = \frac{(s+0.04713)}{(s+0.01)}$
  - which means the velocity error constant will be improved by $10/(\frac{6.793}{3.201}) = 4.713$
- The lag-lead compensated open loop system is:

$$G_{LLC}(s) = \frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$$

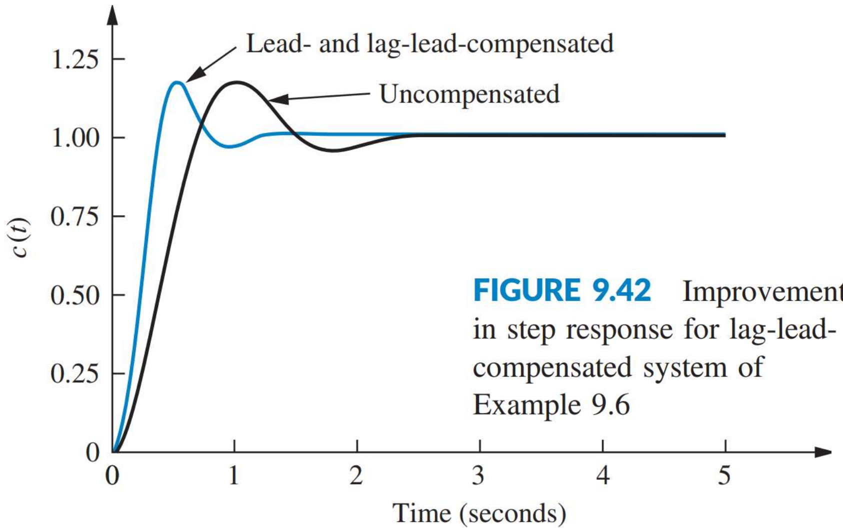

FIGURE 9.42 Improvement in step response for lag-lead-compensated system of Example 9.6

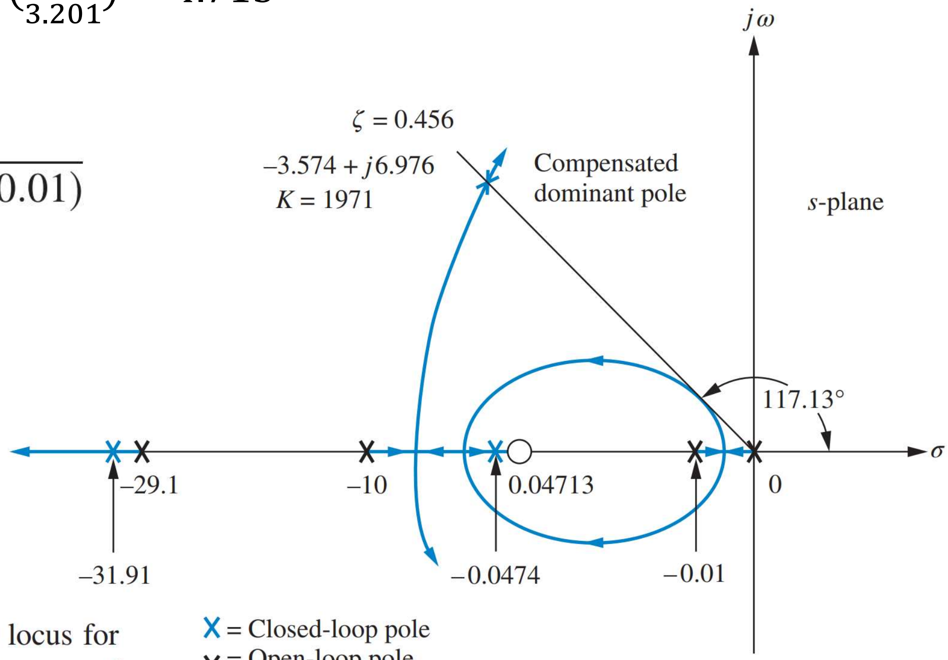

FIGURE 9.41 Root locus for lag-lead-compensated system of Example 9.6

Lag-Lead Compensation

- Finally, the ramp response is examined as follows.

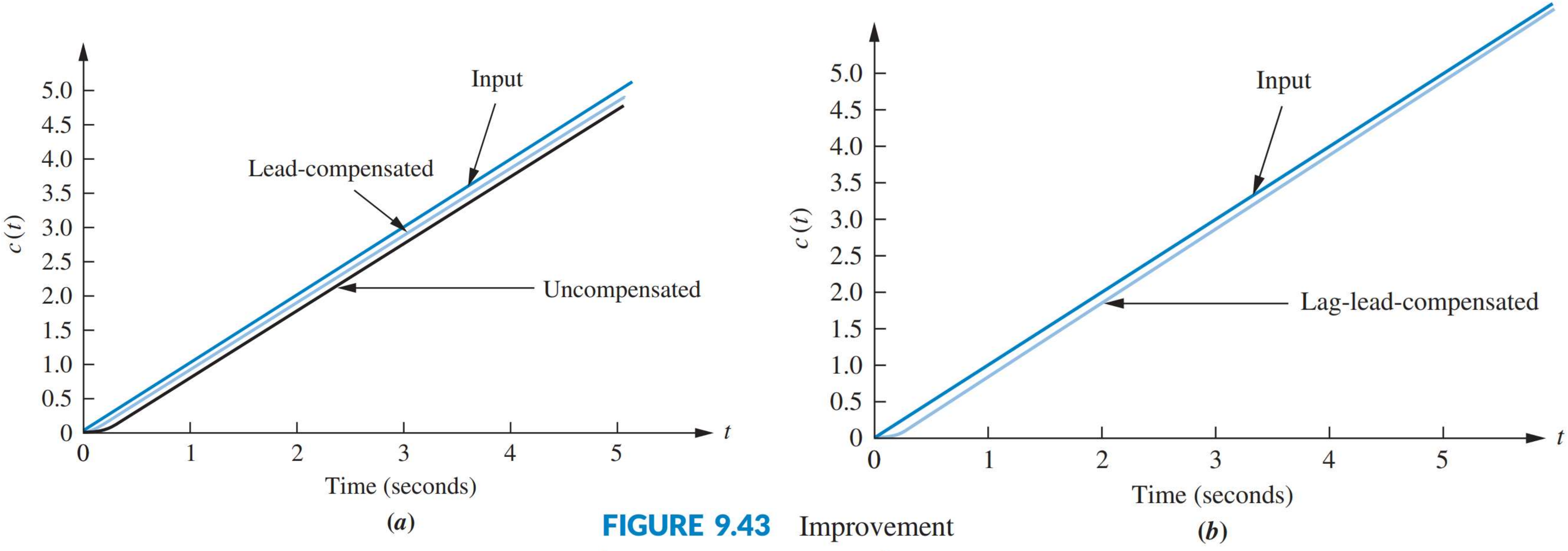

FIGURE 9.43 Improvement in ramp response error for the system of Example 9.6:
a. lead-compensated;
b. lag-lead-compensated

Notch Filter

Notch Filter

- If a plant, such as a mechanical system, has high-frequency vibration modes, then a desired closed-loop response may be difficult to obtain. These high-frequency vibration modes can be modeled as part of the plant's transfer function by pairs of complex poles near the imaginary axis. In a closed-loop configuration, these poles can move closer to the imaginary axis or even cross into the right half-plane.

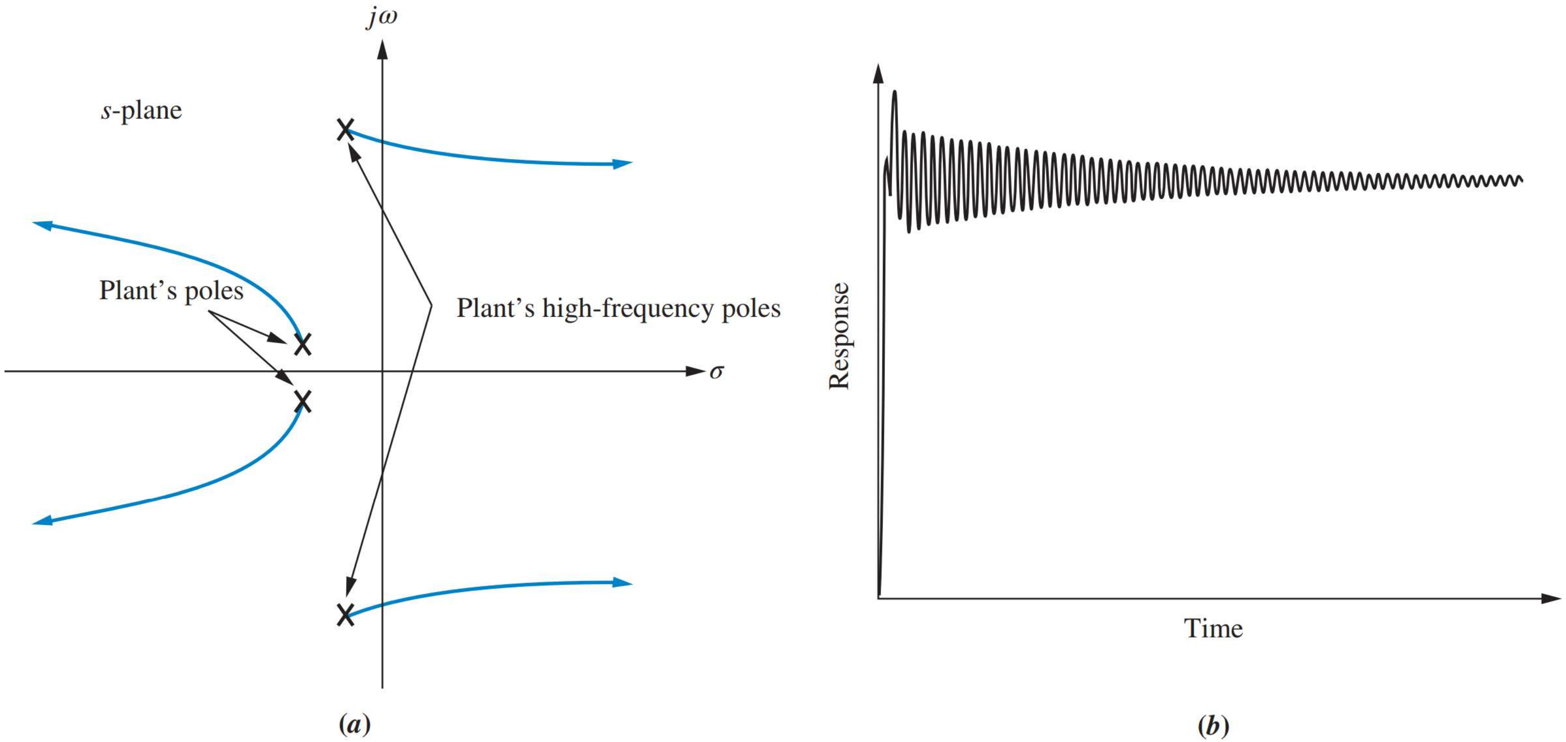

FIGURE 9.44 **a.** Root locus before cascading notch filter; **b.** typical closed-loop step response

Notch Filter

- A notch filter is a special lag-lead compensator can resolve this issue by adding a pair of complex zeros near the dominant open loop poles.

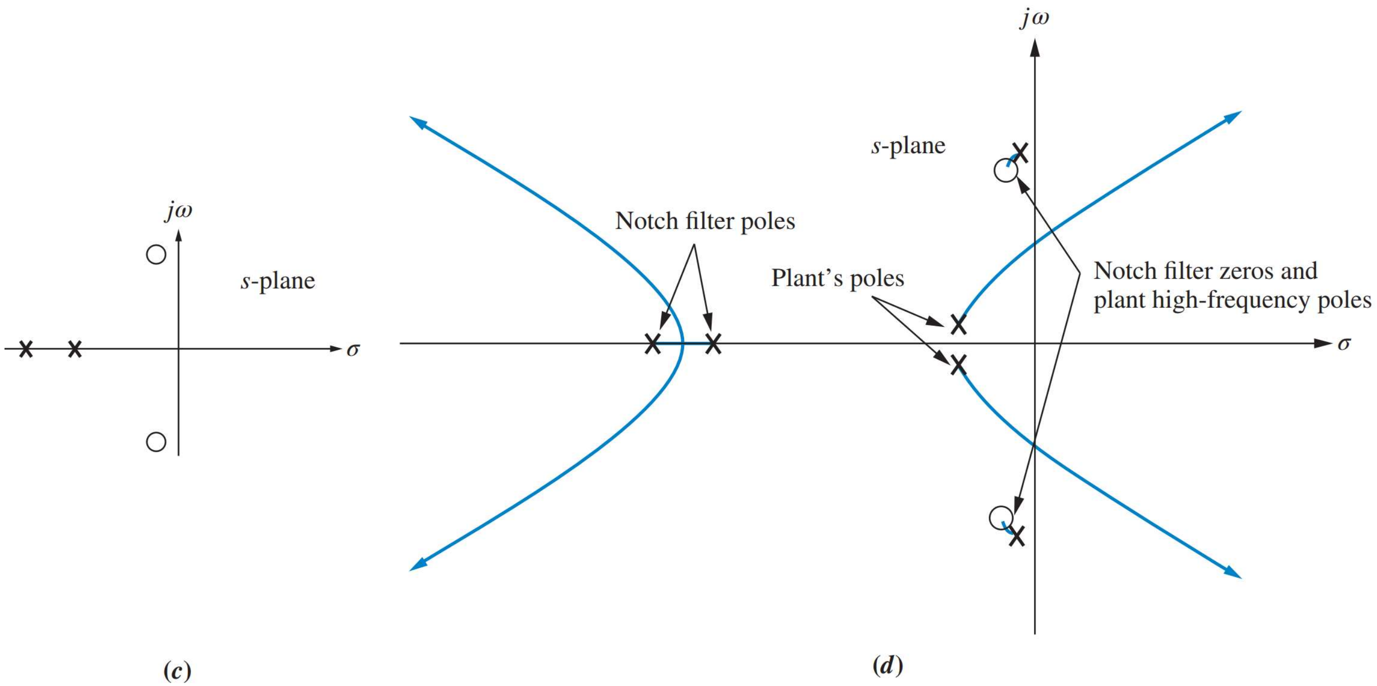

FIGURE 9.44 **a.** Root locus before cascading notch filter; **b.** typical closed-loop step response before cascading notch filter; **c.** pole-zero plot of a notch filter; **d.** root locus after cascading notch filter; (*figure continues*)

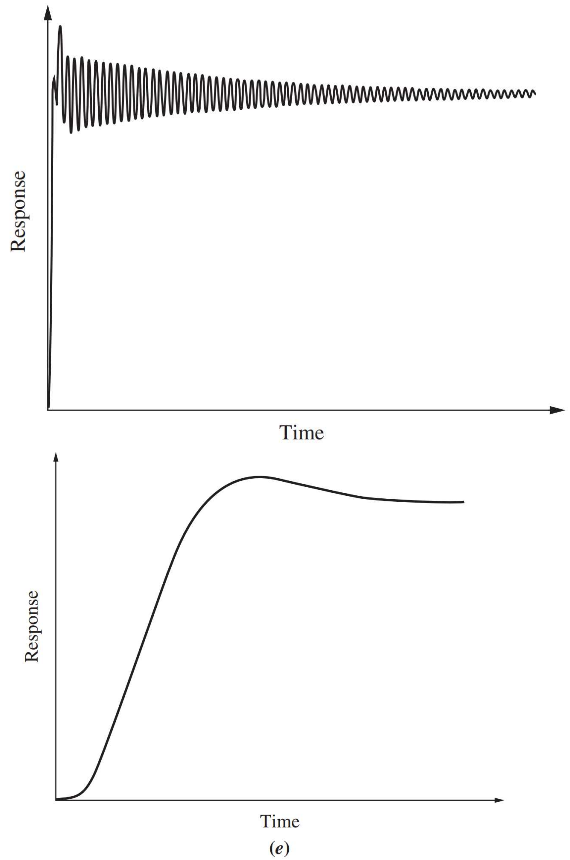

FIGURE 9.44 (*Continued*) **e.** closed-loop step response after cascading notch filter

Notch Filter

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PROBLEM: A unity feedback system with forward transfer function

$$G(s) = \frac{K}{s(s+7)}$$

is operating with a closed-loop step response that has 20% overshoot. Do the following:

a. Evaluate the settling time.

b. Evaluate the steady-state error for a unit ramp input.

c. Design a lag-lead compensator to decrease the settling time by 2 times and decrease the steady-state error for a unit ramp input by 10 times. Place the lead zero at -3.

ANSWERS:

a.

b.

c.

So far, we have discussed all Cascaded Compensators

Summary

- PI and Lag
- PD and Lead
- PID and Lag-lead

TABLE 9.7 Types of cascade compensators

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K\frac{s+z_c}{s}$	**1.** Increases system type. **2.** Error becomes zero. **3.** Zero at $-z_c$ is small and negative. **4.** Active circuits are required to implement.
Improve steady-state error	Lag	$K\frac{s+z_c}{s+p_c}$	**1.** Error is improved but not driven to zero. **2.** Pole at $-p_c$ is small and negative. **3.** Zero at $-z_c$ is close to, and to the left of, the pole at $-p_c$. **4.** Active circuits are not required to implement.
Improve transient response	PD	$K(s+z_c)$	**1.** Zero at $-z_c$ is selected to put design point on root locus. **2.** Active circuits are required to implement. **3.** Can cause noise and saturation; implement with rate feedback or with a pole (lead).
Improve transient response	Lead	$K\frac{s+z_c}{s+p_c}$	**1.** Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus. **2.** Pole at $-p_c$ is more negative than zero at $-z_c$. **3.** Active circuits are not required to implement.
Improve steady-state error and transient response	PID	$K\frac{(s+z_{lag})(s+z_{lead})}{s}$	**1.** Lag zero at $-z_{lag}$ and pole at origin improve steady-state error. **2.** Lead zero at $-z_{lead}$ improves transient response. **3.** Lag zero at $-z_{lag}$ is close to, and to the left of, the origin. **4.** Lead zero at $-z_{lead}$ is selected to put design point on root locus. **5.** Active circuits required to implement. **6.** Can cause noise and saturation; implement with rate feedback or with an additional pole.
Improve steady-state error and transient response	Lag-lead	$K\frac{(s+z_{lag})(s+z_{lead})}{(s+p_{lag})(s+p_{lead})}$	**1.** Lag pole at $-p_{lag}$ and lag zero at $-z_{lag}$ are used to improve steady-state error. **2.** Lead pole at $-p_{lead}$ and lead zero at $-z_{lead}$ are used to improve transient response. **3.** Lag pole at $-p_{lag}$ is small and negative. **4.** Lag zero at $-z_{lag}$ is close to, and to the left of, lag pole at $-p_{lag}$. **5.** Lead zero at $-z_{lead}$ and lead pole at $-p_{lead}$ are selected to put design point on root locus. **6.** Lead pole at $-p_{lead}$ is more negative than lead zero at $-z_{lead}$. **7.** Active circuits are not required to implement.

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Feedback Compensation

Minor loop or inner loop

- For system with more than one feedback loop, typically having more than one tuning parameter, one way to deal with it is to first reduce the number of the loop and then design via generalized root locus.

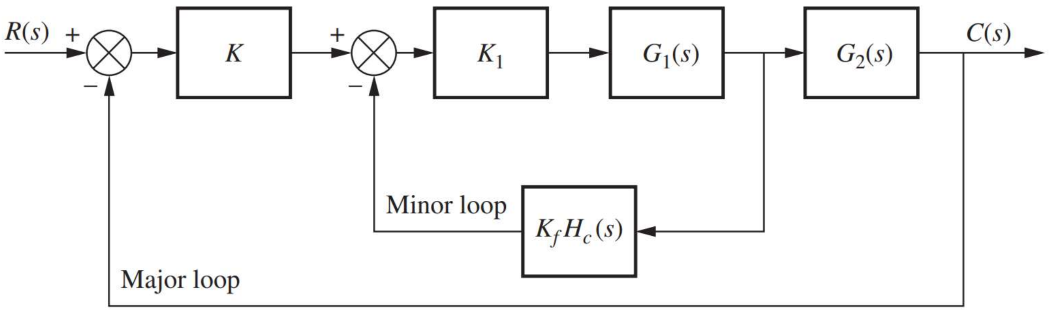


FIGURE 9.45 Generic control system with feedback compensation.

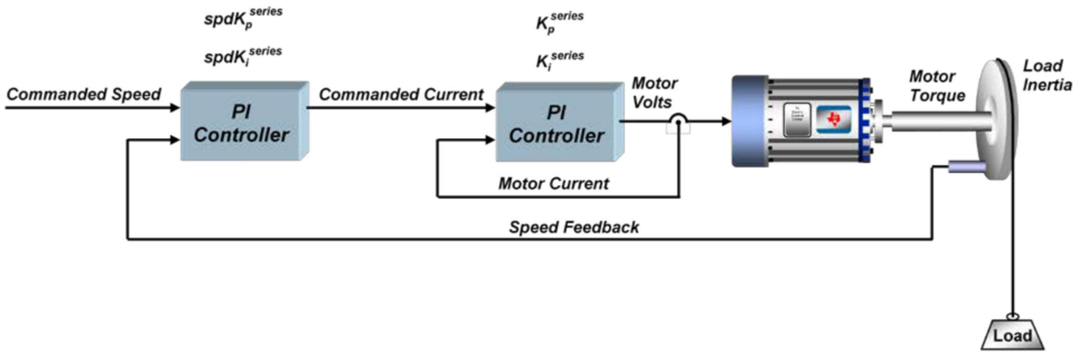


Figure 11-6. Cascaded Speed Control Loop

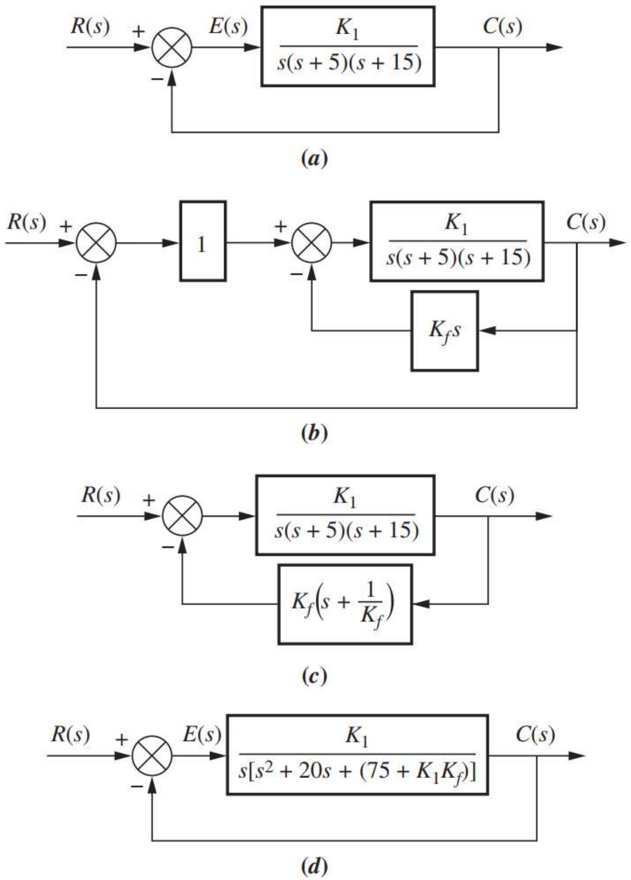
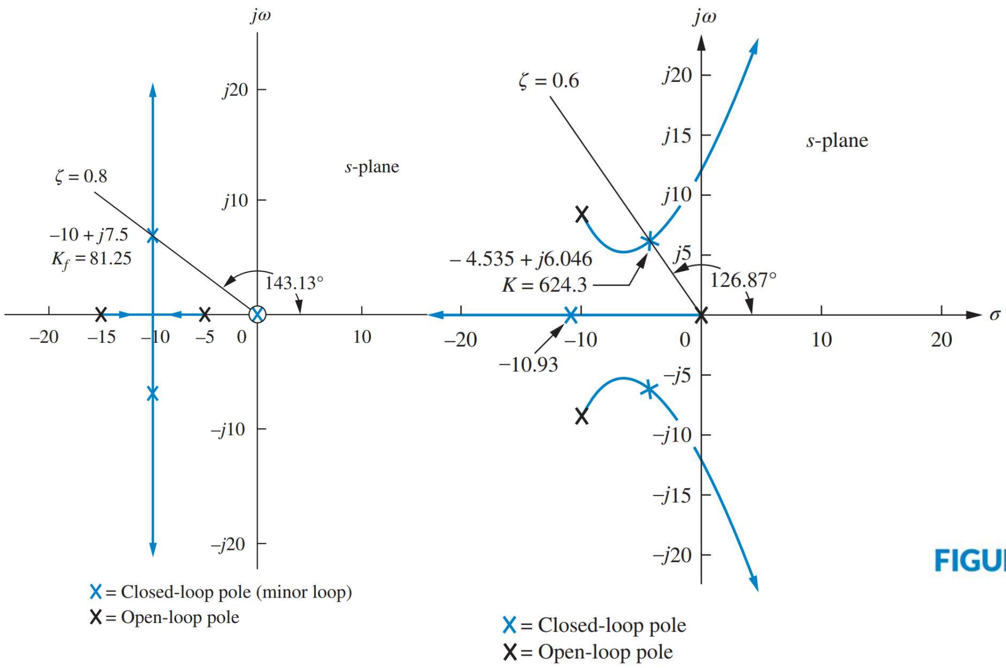


FIGURE 9.49 **a.** System for Example 9.7; **b.** system with rate feedback compensation; **c.** equivalent compensated system; **d.** equivalent compensated system showing unity feedback

Minor loop or inner loop

- Alternatively, we can design the minor loop first.
  - This will often simplify the design process because we are often dealing a lower order system in minor loop.



Minor-Loop Feedback Compensation

PROBLEM: For the system of Figure 9.55(*a*), design minor-loop feedback compensation, as shown in Figure 9.55(*b*), to yield a damping ratio of 0.8 for the minor loop and a damping ratio of 0.6 for the closed-loop system.

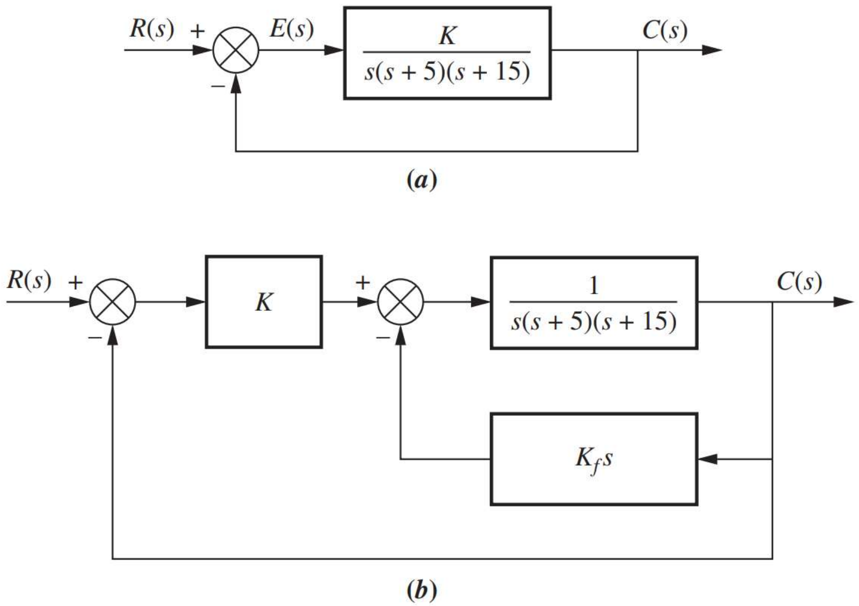


FIGURE 9.55 **a.** Uncompensated system and **b.** feedback-compensated system for Example 9.8

Minor loop or inner loop

PROBLEM: For the system of Figure 9.59, design minor-loop rate feedback compensation to yield a damping ratio of 0.7 for the minor loop's dominant poles and a damping ratio of 0.5 for the closed-loop system's dominant poles.

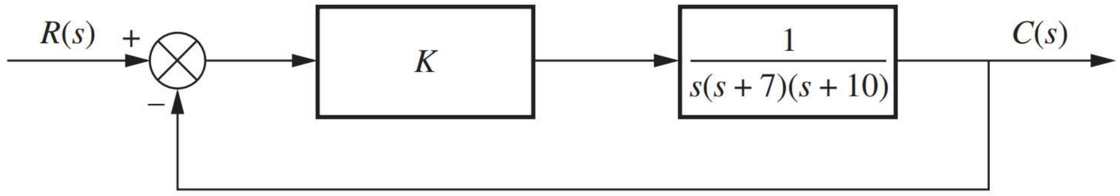
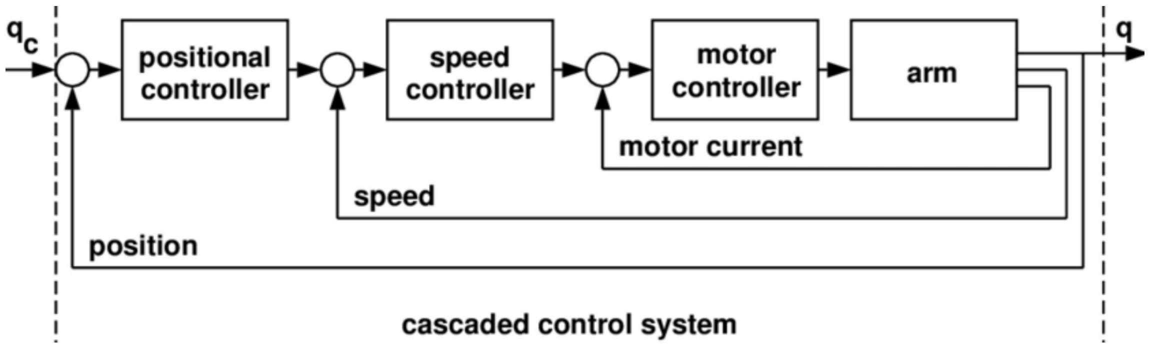


FIGURE 9.59 System for Skill-Assessment Exercise 9.4

Cascaded Loops

- The minor loop concept can be extended to even more loops.
  - For example, the position-velocity-torque control system in a servo system.



Cascaded Loops

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- However, three loops are not necessarily to be the standard, e.g., Texas Instruments (TI) skips the velocity loop during position control mode.
  - *The position loop uses a PD control, which is equivalent to a position proportional control outer loop plus a velocity control inner loop.*

An Overview of InstaSPIN-MOTION and SpinTAC www.ti.com

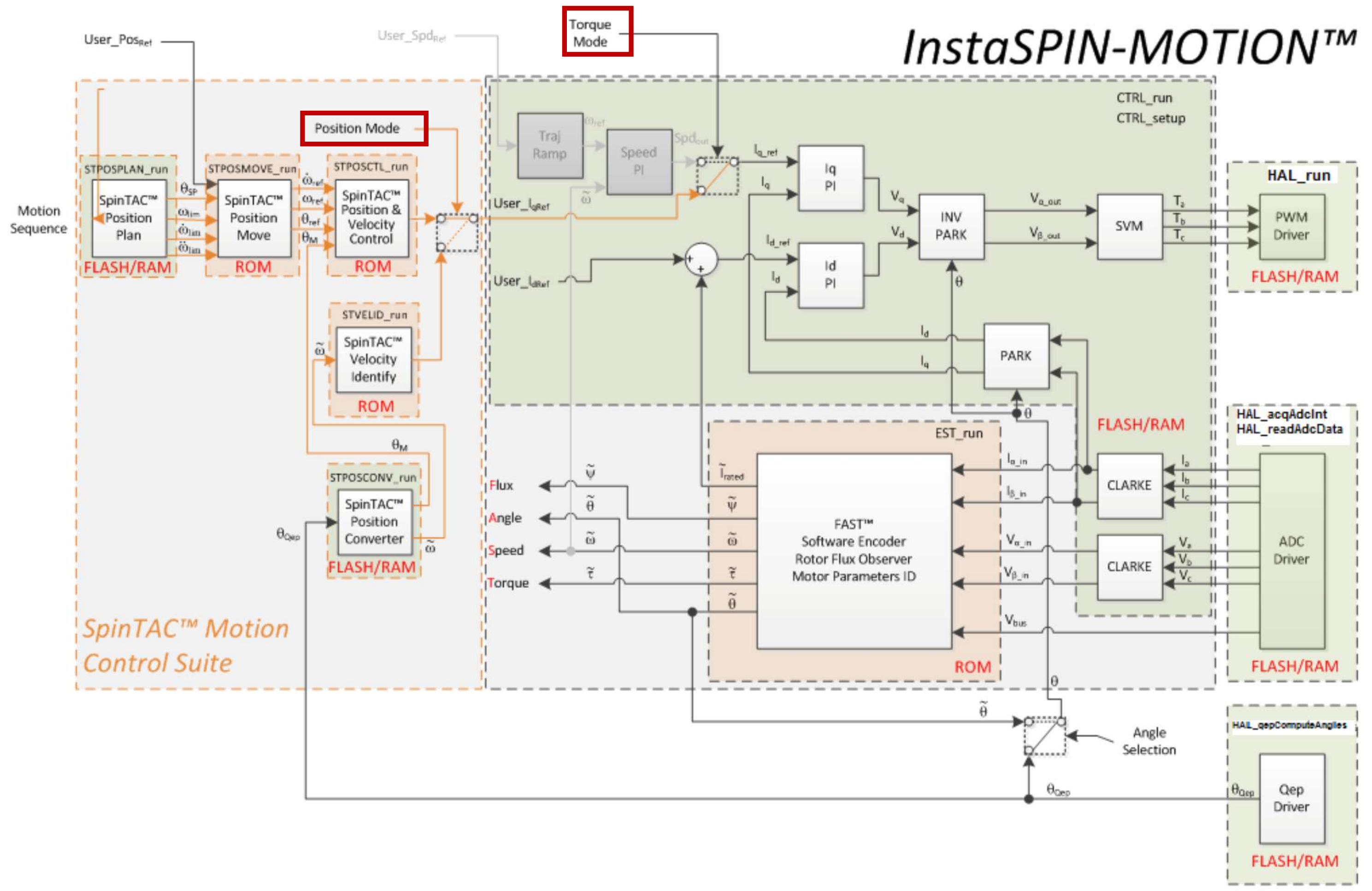

Figure 1-14. InstaSPIN-MOTION Position Control with Mechanical Sensor and Redundant FAST Software Sensor

Teaching Old Motors New Tricks - Part 1
https://www.youtube.com/watch?v=fpTvZlnrsP0