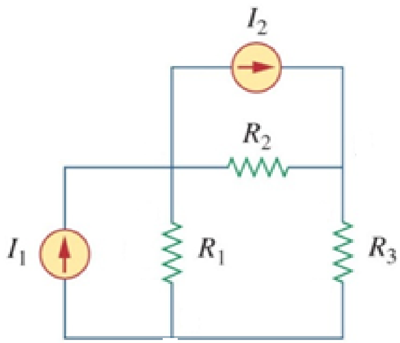
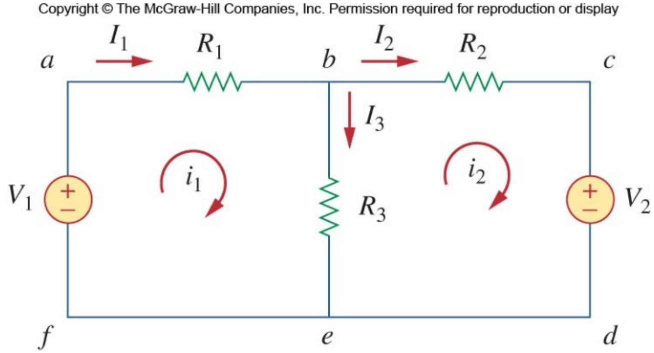
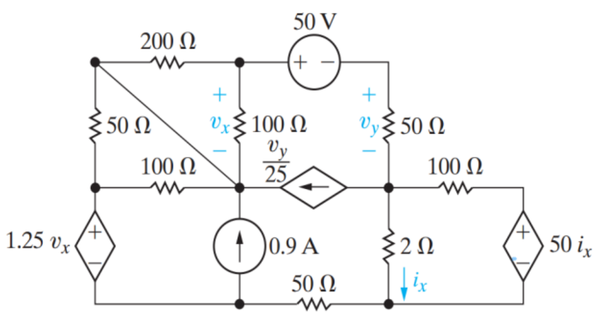
Lecture 5
- RC/RL First-Order Circuits

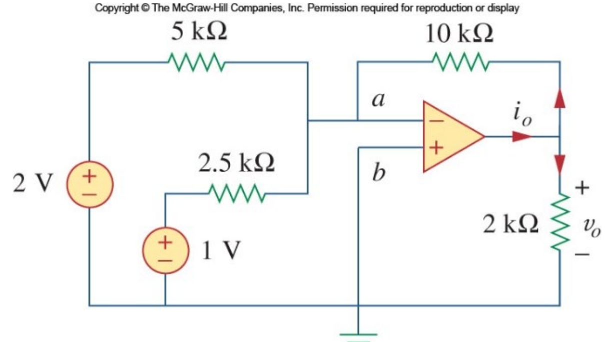
Beginning of Temporal Behavior Analysis of Circuits

- Till now we discussed static analysis of a circuit
  - Responses at a given time depend only on inputs at that time.
  - Circuit responds to input changes infinitely fast.







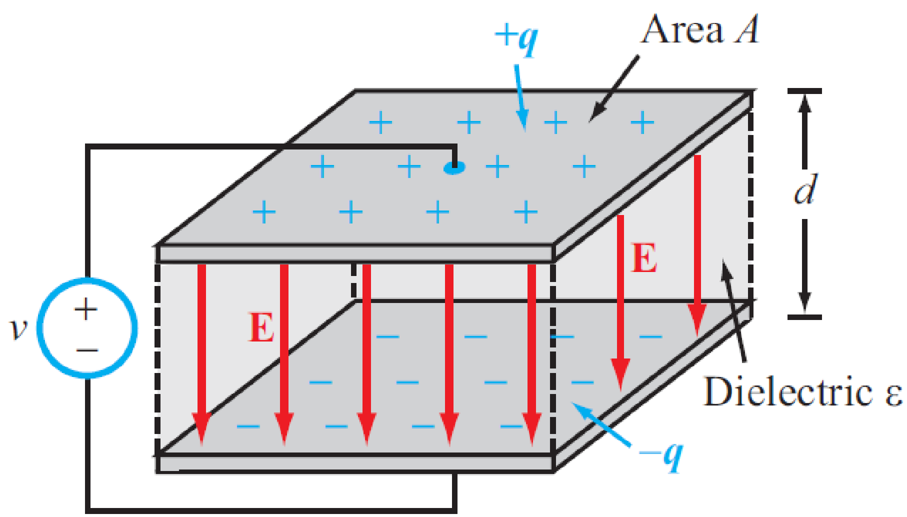


Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

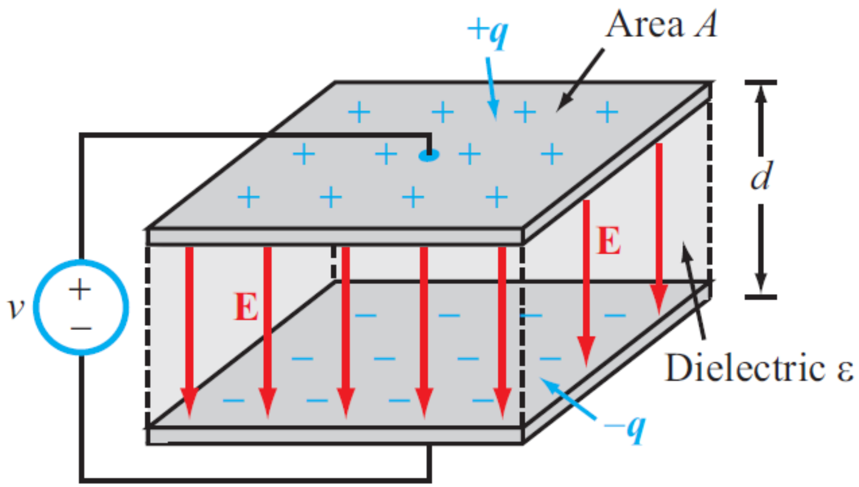
Capacitors

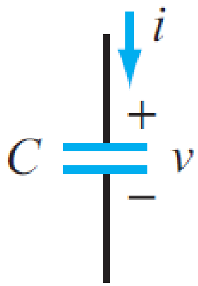
Storage element that stores energy in electric field



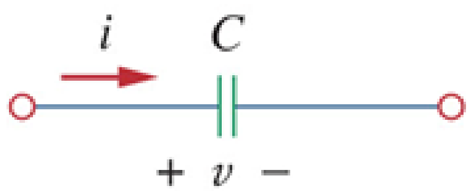
Parallel plate capacitor

V-I Relationship of Capacitors





Stored Energy

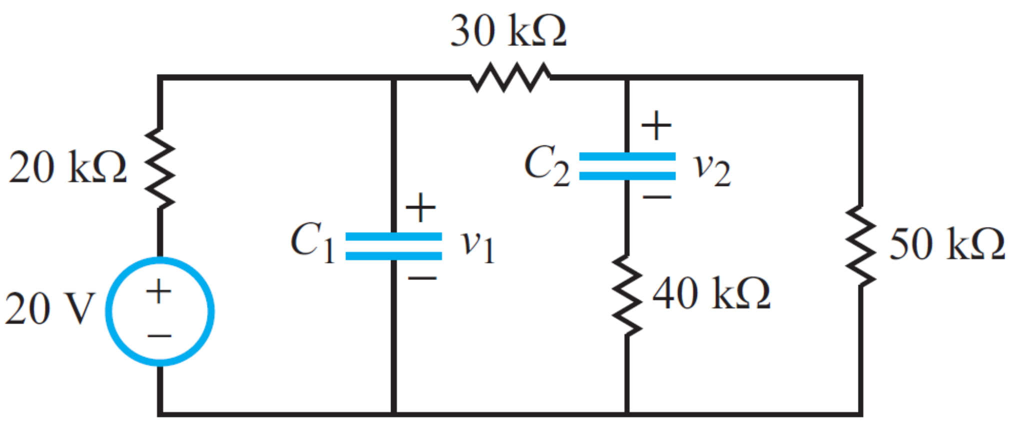


$$i = C\frac{dv}{dt}$$

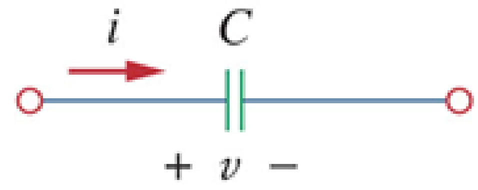
- The instantaneous power delivered to the capacitor is

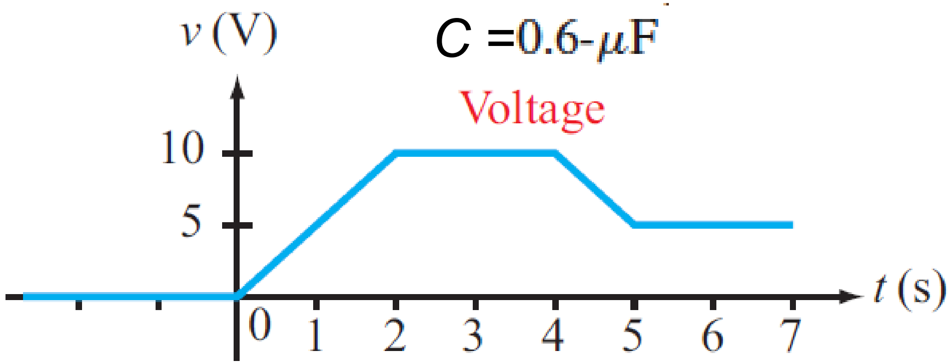
- The energy stored in a capacitor is:

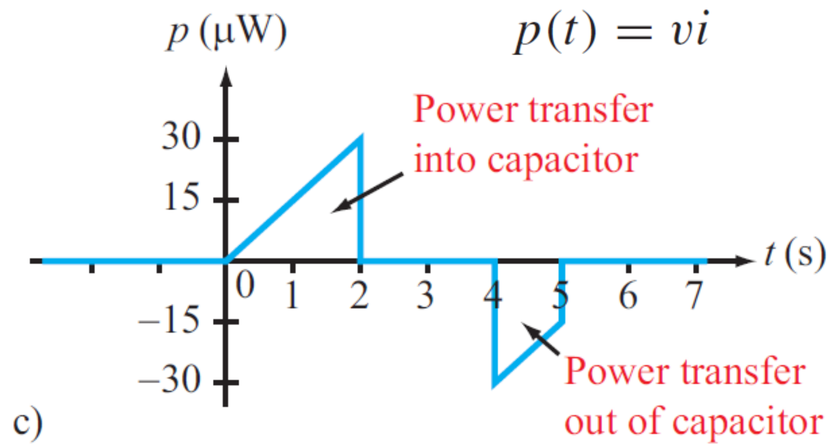
Example-1

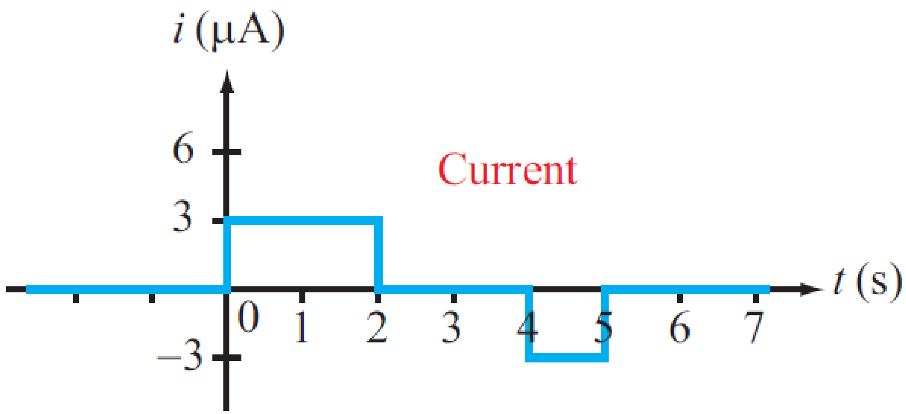


Example-2 Capacitor Response

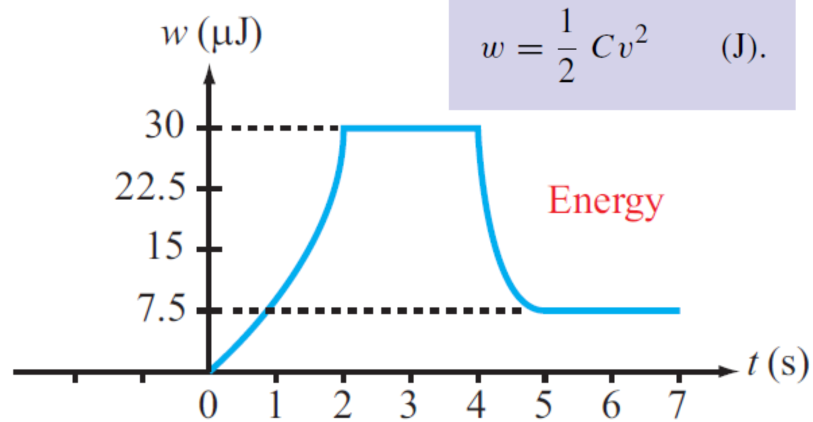




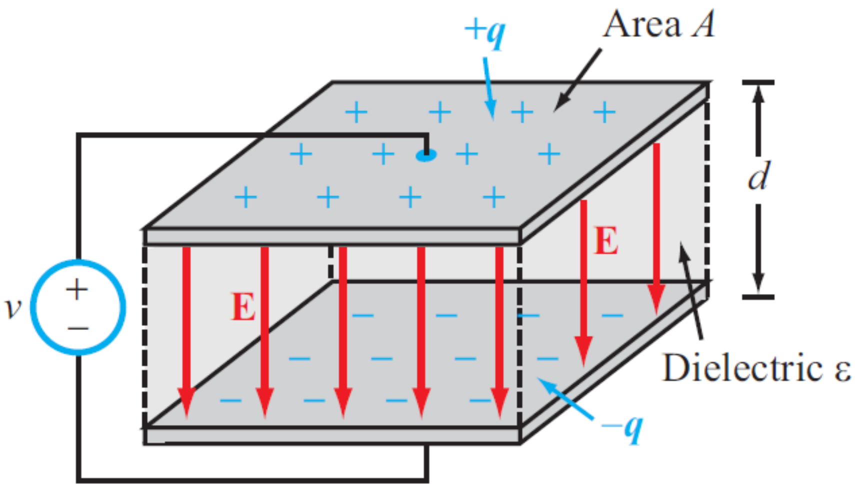




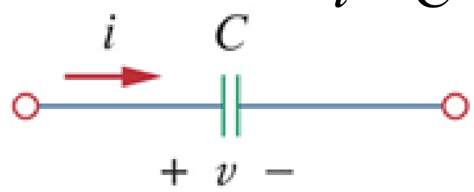
$$w = \frac{1}{2} C v^2 \quad \text{(J).}$$

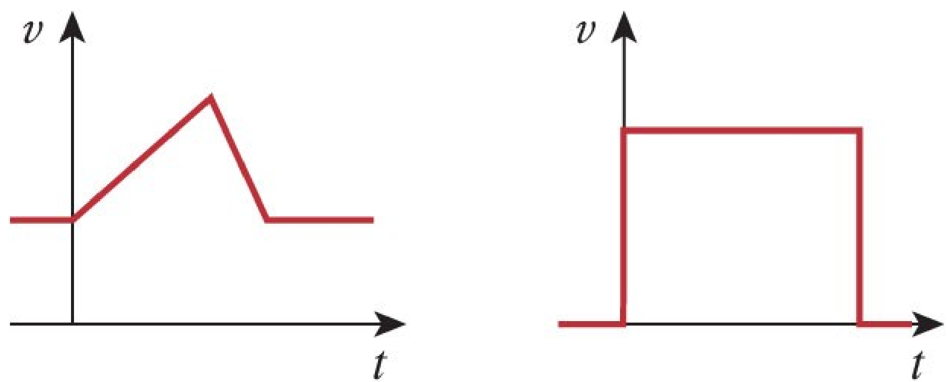


Important Property of Capacitors



$$i = C\frac{dv}{dt}$$

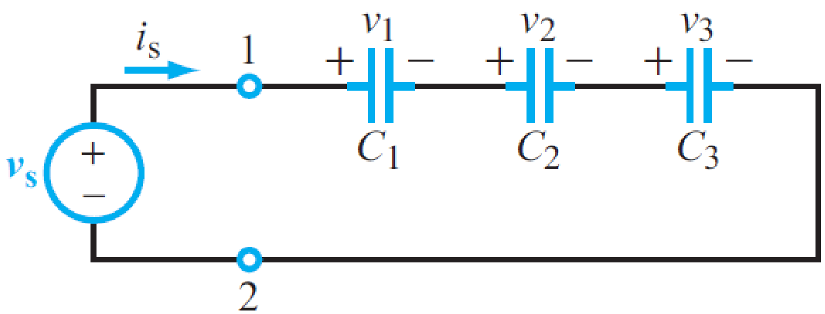


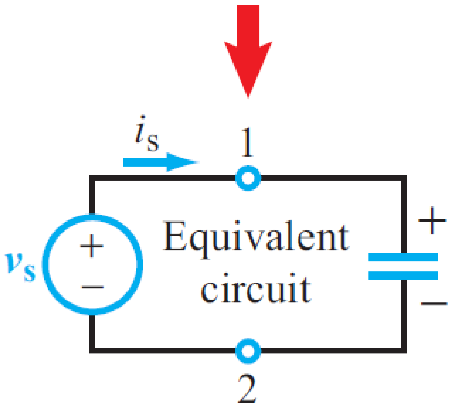


(a) (b)

Capacitors in Series

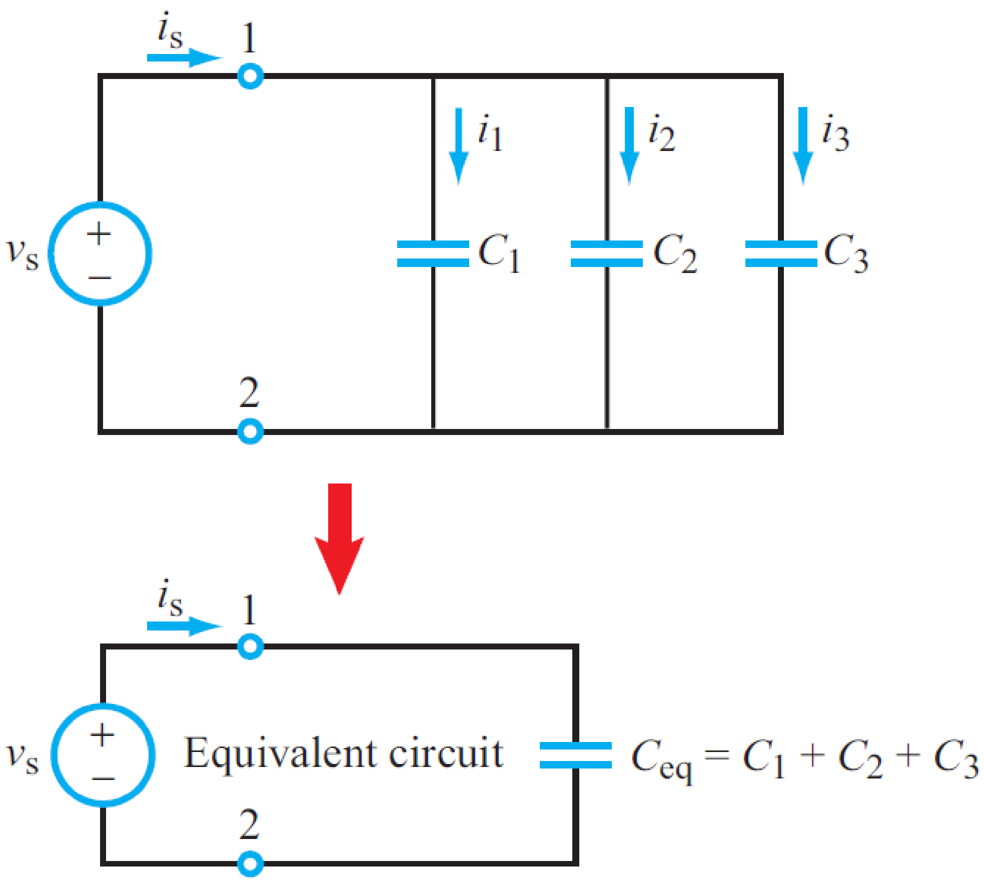
Combining In-Series Capacitors





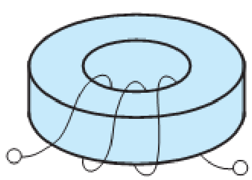
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

Capacitors in Parallel

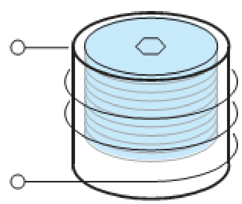


Inductors

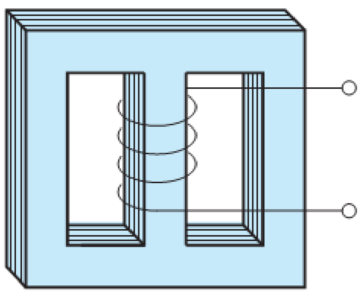
- A storage element that stores energy in magnetic field.
  - They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



(a) Toroidal inductor

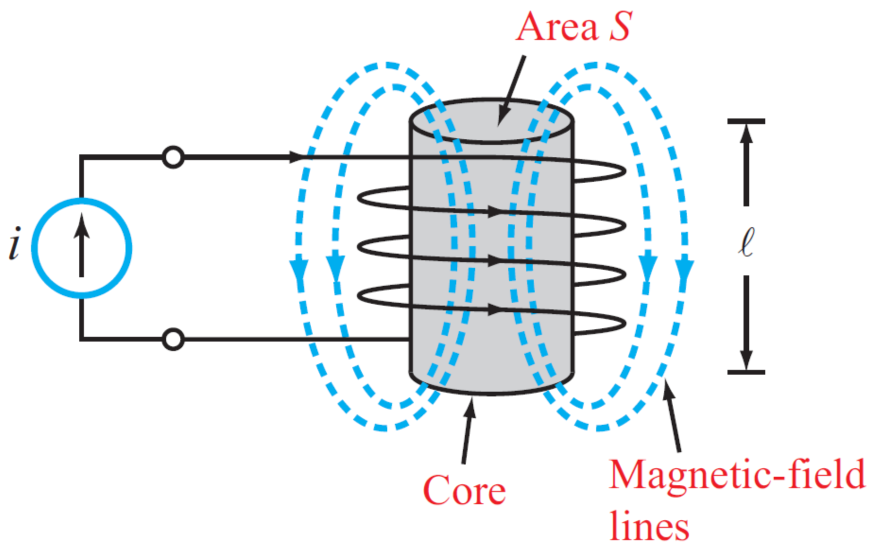


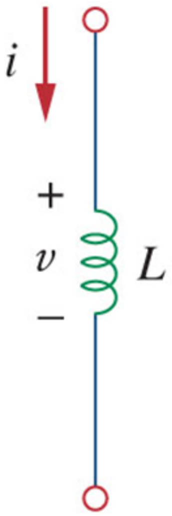
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance



(c) Inductor with a laminated iron core

V-I Relationship of Inductors





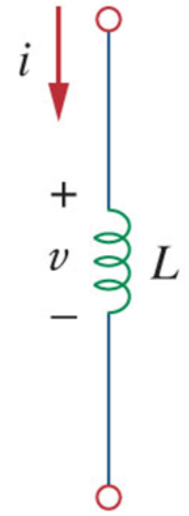
$$v = L\frac{di}{dt}$$

[Source: Berkeley]

Energy Stored in an Inductor

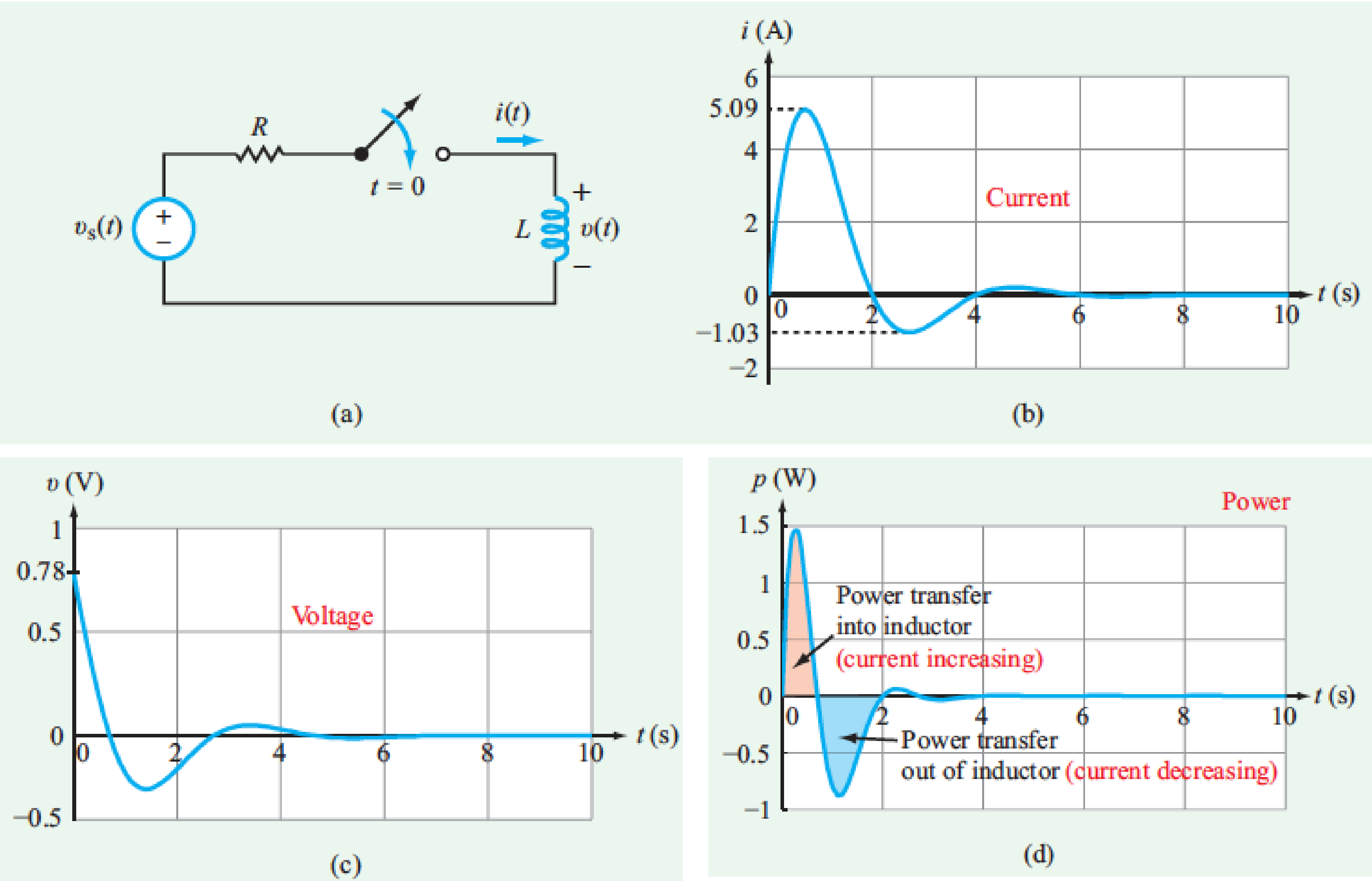
- The power delivered to the inductor is:

- The energy stored is:

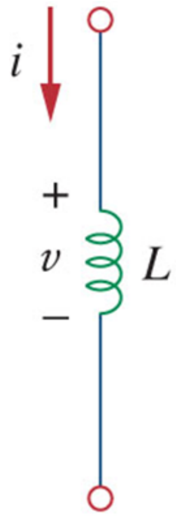


$$v = L\frac{di}{dt}$$

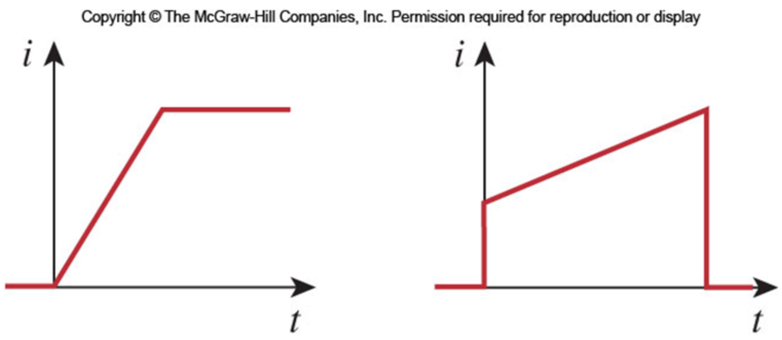
Inductor Response



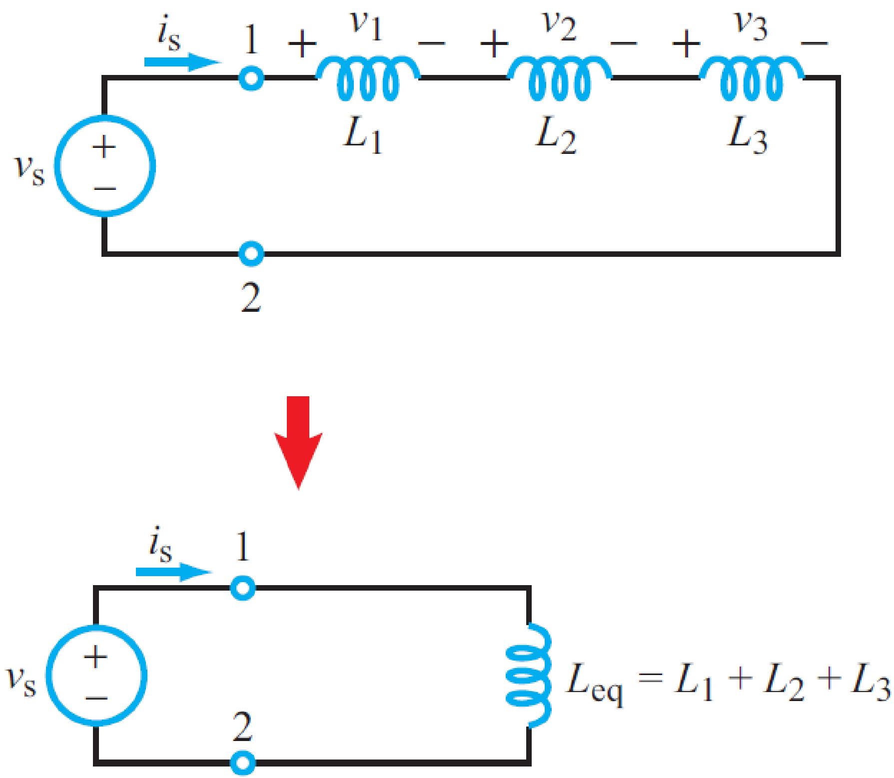
Important Property of Inductors



$$v = L\frac{di}{dt}$$

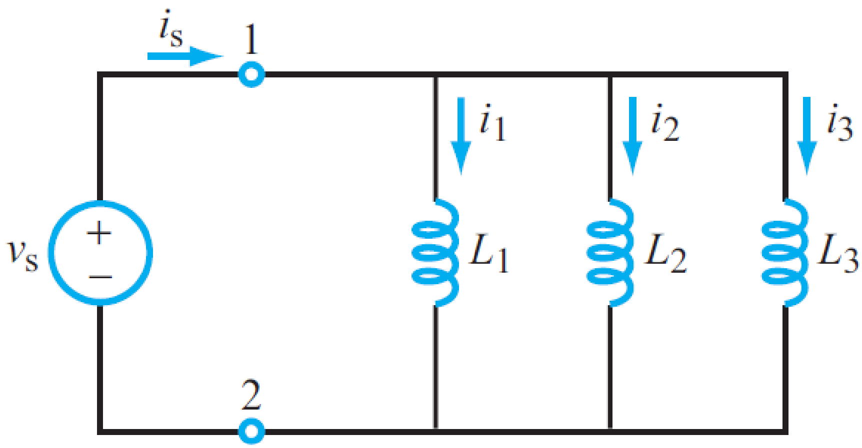


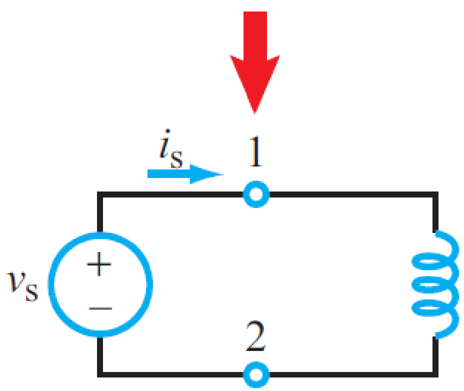
Inductors in Series



Inductors in Parallel

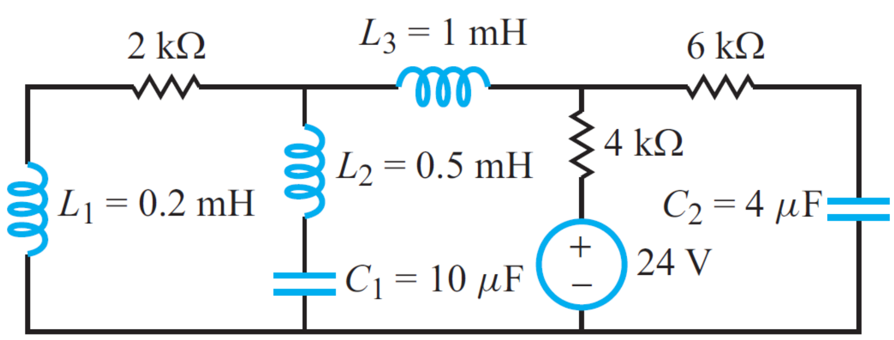
Combining In-Parallel Inductors

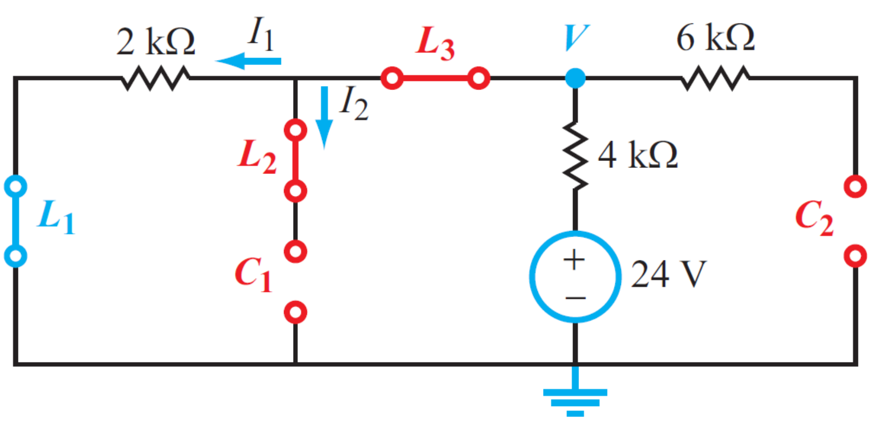




$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

Example


2 kΩ
$L_3 = 1$ mH
6 kΩ
4 kΩ
$L_2 = 0.5$ mH
$L_1 = 0.2$ mH
$C_2 = 4\ \mu$F
$C_1 = 10\ \mu$F
+
–
24 V


2 kΩ
I_1
L_3
V
6 kΩ
I_2
L_2
4 kΩ
L_1
C_2
C_1
+
–
24 V

Summary of Resistors, Capacitors and Inductors

Table 5-4: Basic properties of R, L, and C.

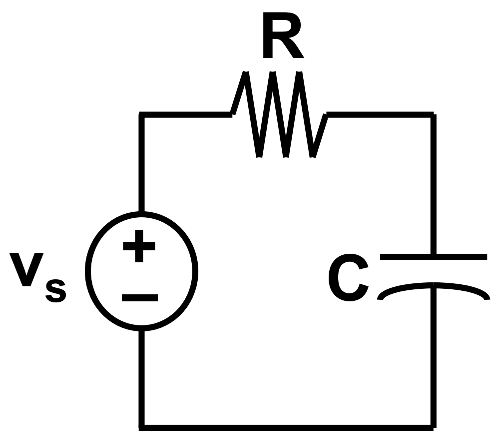
Property	R	L	C
i–υ relation	$i = \frac{\upsilon}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} \upsilon\, dt' + i(t_0)$	$i = C\frac{d\upsilon}{dt}$
υ-i relation	$\upsilon = iR$	$\upsilon = L\frac{di}{dt}$	$\upsilon = \frac{1}{C}\int_{t_0}^{t} i\, dt' + \upsilon(t_0)$
p (power transfer in)	$p = i^2R$	$p = Li\frac{di}{dt}$	$p = C\upsilon\frac{d\upsilon}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}C\upsilon^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can υ change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

Outline

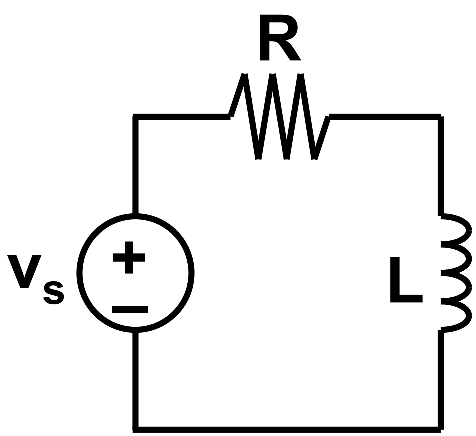
- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

RC and RL Circuits

- A circuit that contains only source(s), resistor(s) and a capacitor is called an ***RC circuit***.



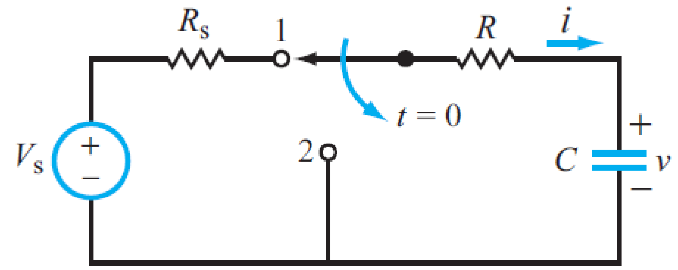
- A circuit that contains only source(s), resistor(s) and an inductor is called an ***RL circuit***.



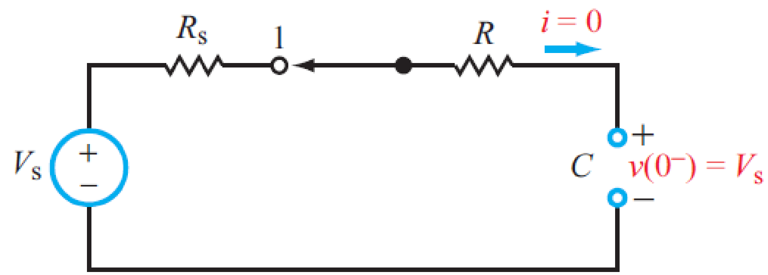
[Source: Berkeley]

Natural Response

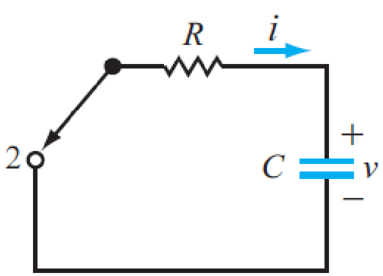
Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing <u>no independent sources</u>).



Natural Response of a Charged Capacitor



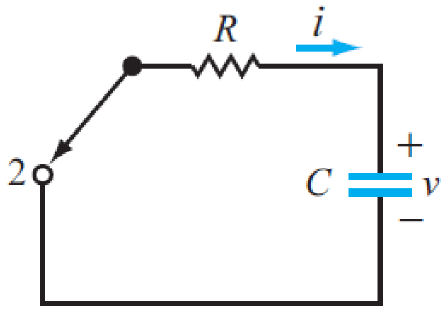
(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2;



(b) $t = 0$ is the instant just after it was moved, $t = 0$ is synonymous with $t = 0^+$.

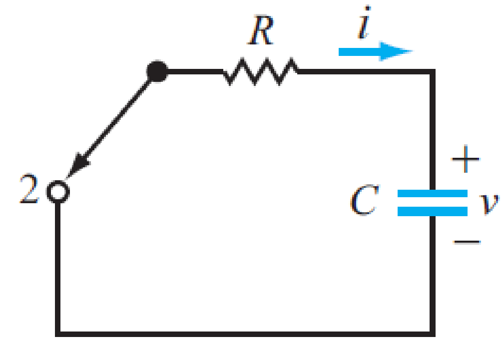
[Source: Berkeley]

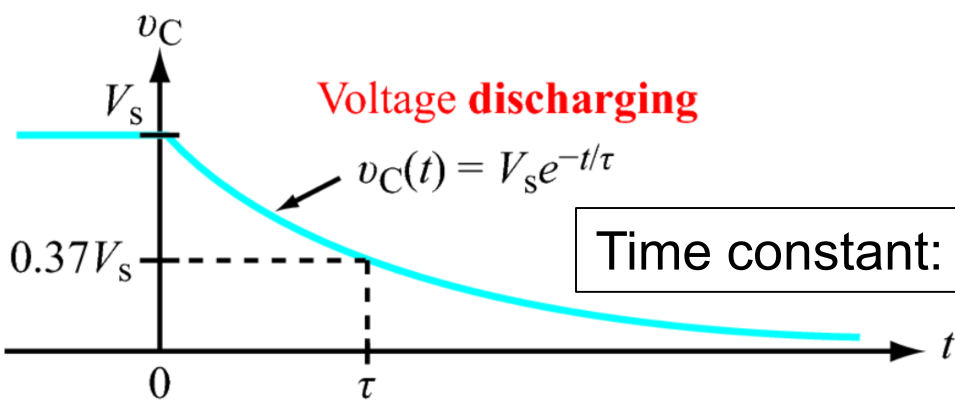
Natural Response of a Charged Capacitor



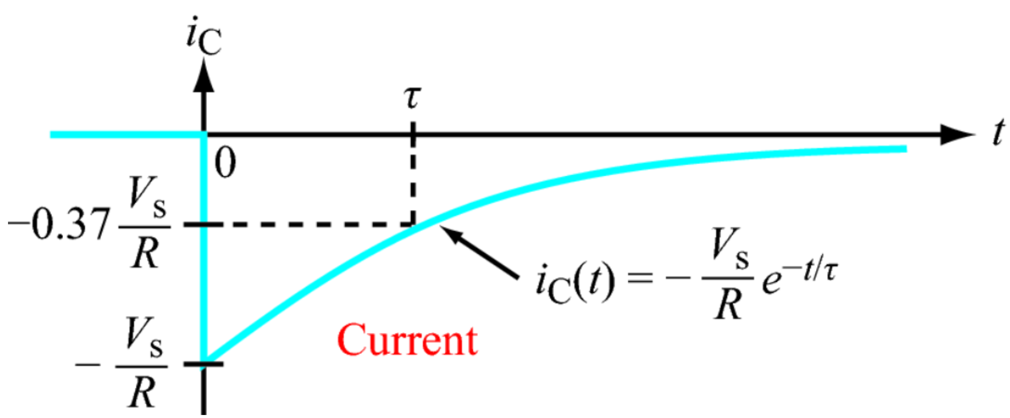
[Source: Berkeley]

Natural Response of RC Circuit



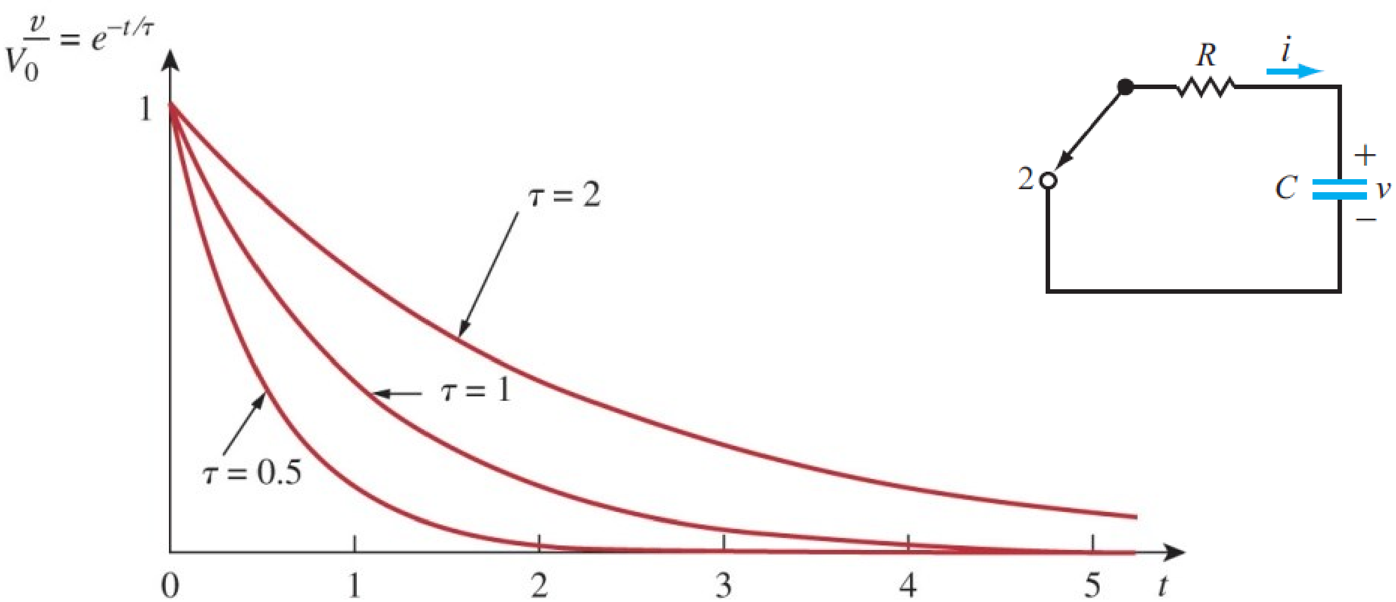


Time constant: $\tau = RC$



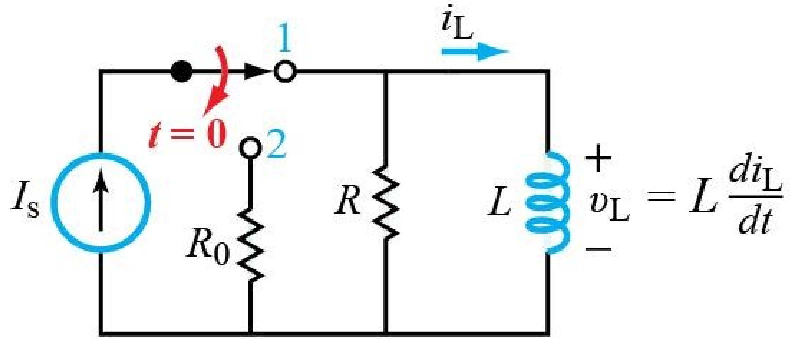
Time Constant τ ($= RC$)

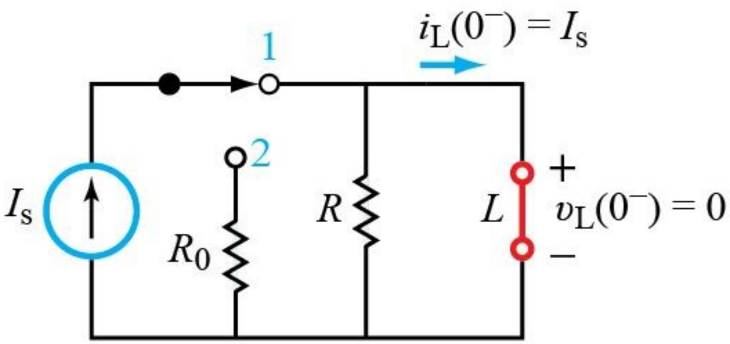
- A circuit with a small time constant has a fast response and vice versa.

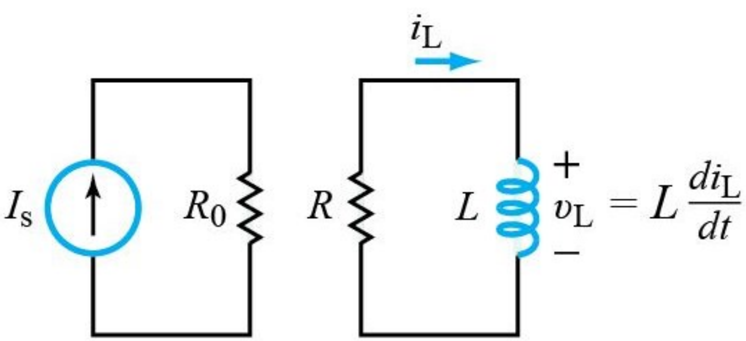


[Source: Berkeley]

Natural Response of the RL Circuit

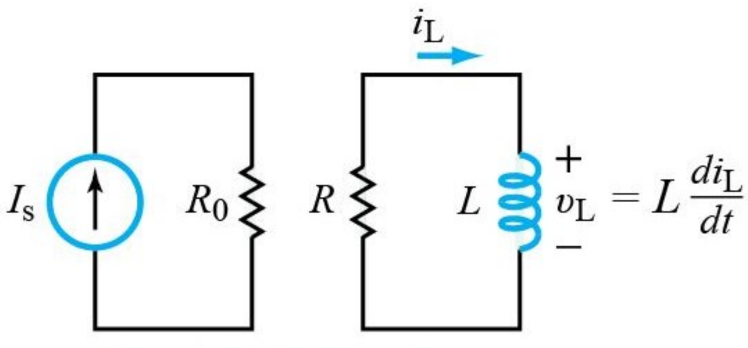






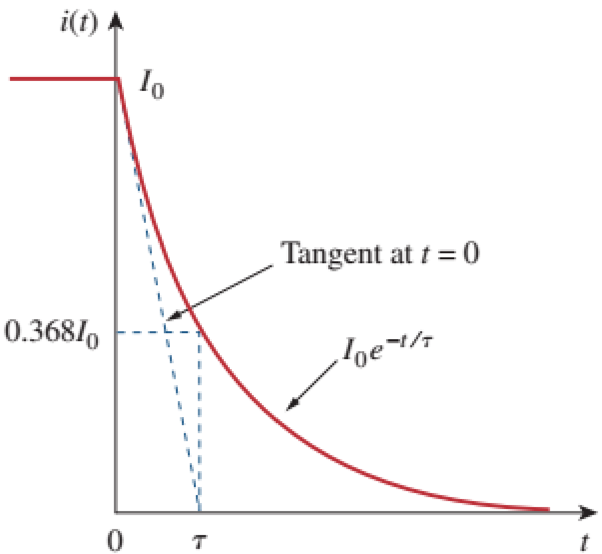
[Source: Berkeley]

Natural Response of the RL Circuit



[Source: Berkeley]

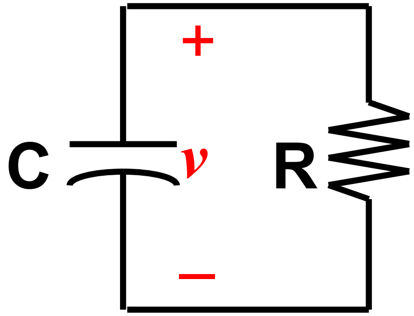
Natural Response of the RL Circuit



[Source: Berkeley]

Natural Response Summary

RC Circuit



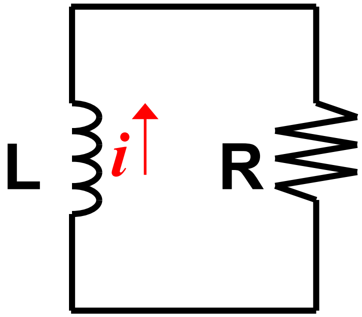
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

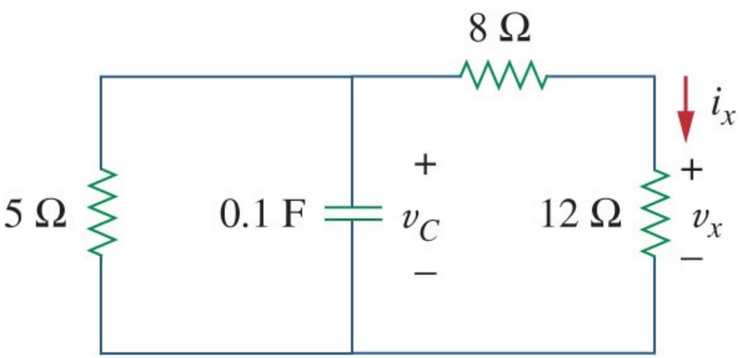
$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \frac{L}{R}$

[Source: Berkeley]

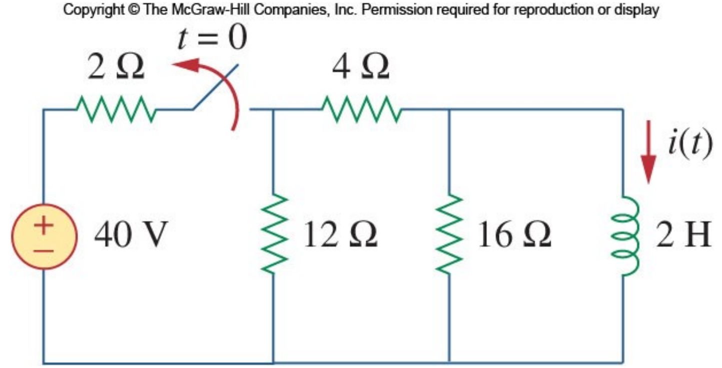
Example

- In the circuit below, let $v_C(t=0) = 15\text{V}$. Find v_C, v_x, and i_x for $t > 0$.

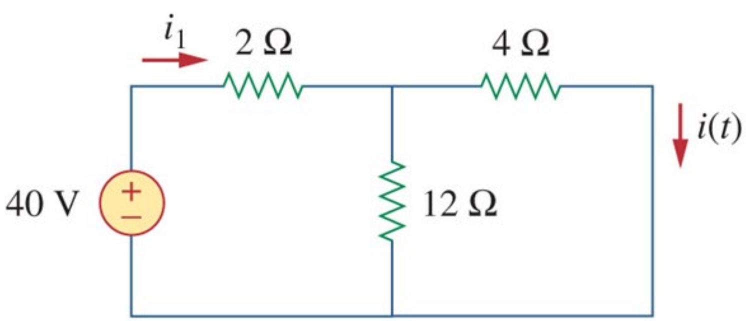


Example

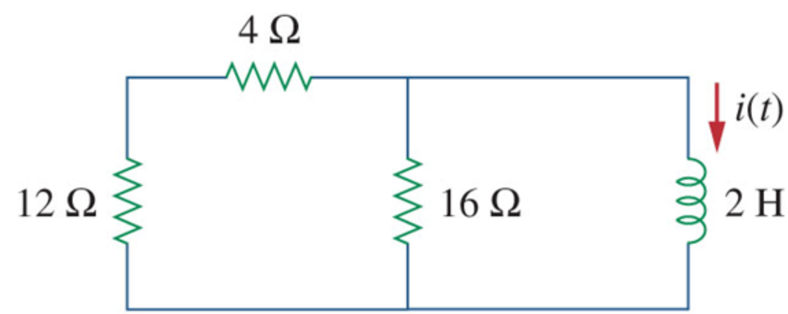
- The switch in the circuit below has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



When $t < 0$



When $t > 0$

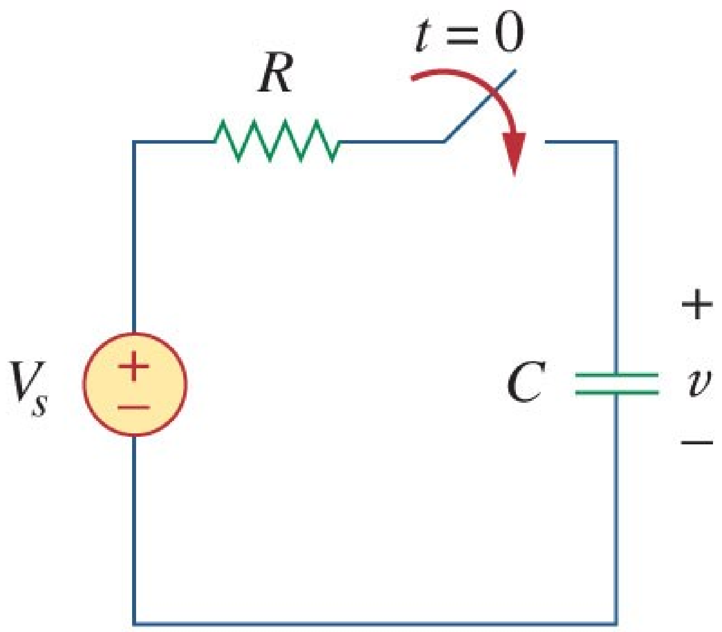


Outline

- Capacitors and inductors
- Natural response of RC/RL circuits
- Step response of RC/RL circuits
- Others

Step Response of RC Circuit

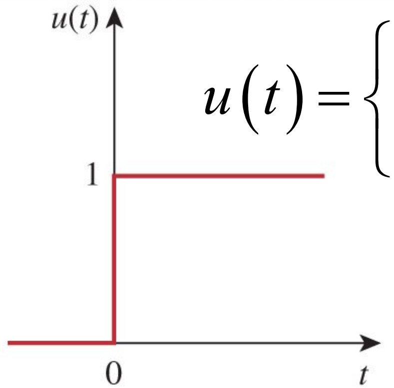
- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.



$$v(0^-) = v(0^+) = v_0$$

The unit step function *u(t)*

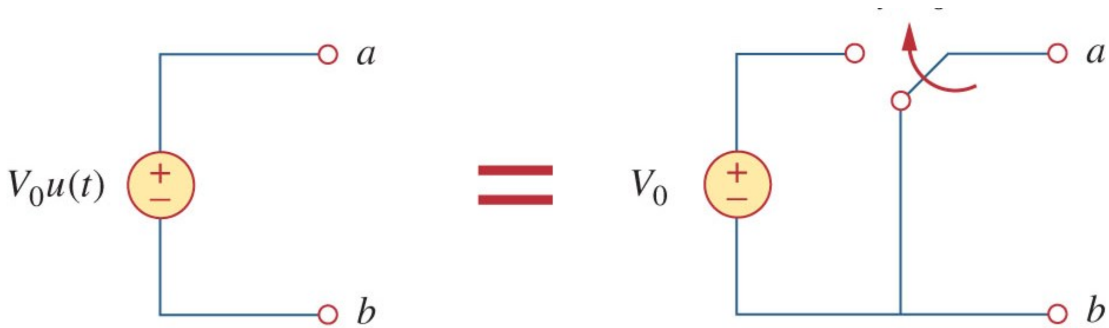
- A step function is one that maintains a constant value before a certain time and then changes to another constant afterwards.



$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$

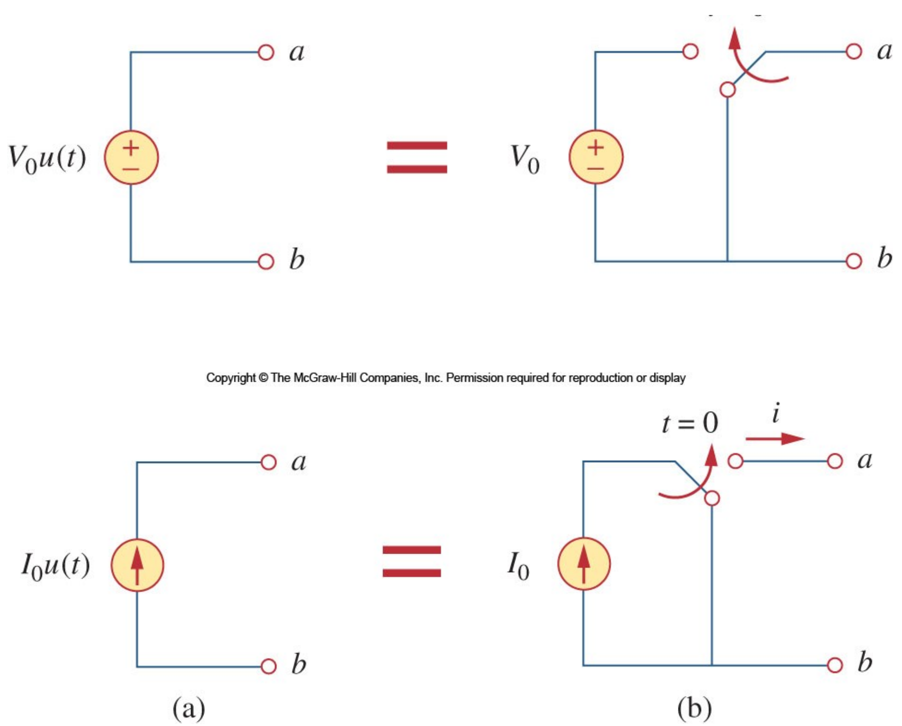
switching time may be shifted to $t = t_0$ by

$$u(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t>t_0\end{cases}$$

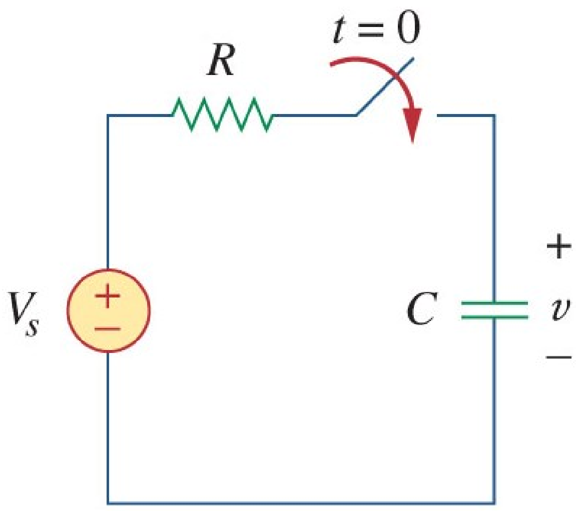


Equivalent Circuit of Unit Step

- The unit step function has an equivalent circuit to represent when it is used to switch on a source.

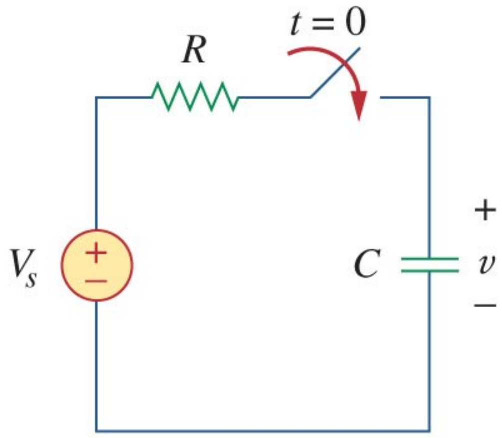


Step Response of the RC Circuit



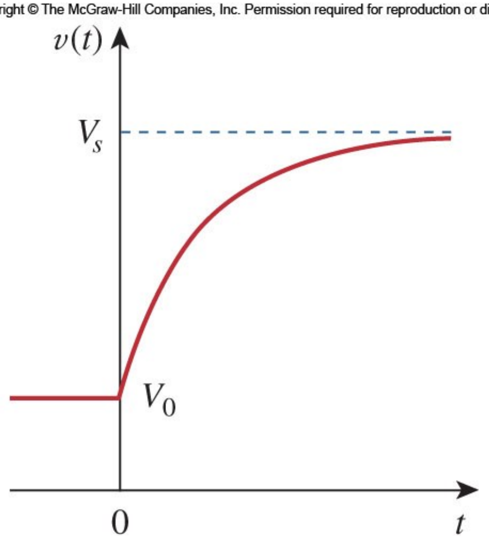
$$v(0^-) = v(0^+) = V_0$$

Step Response of the RC Circuit



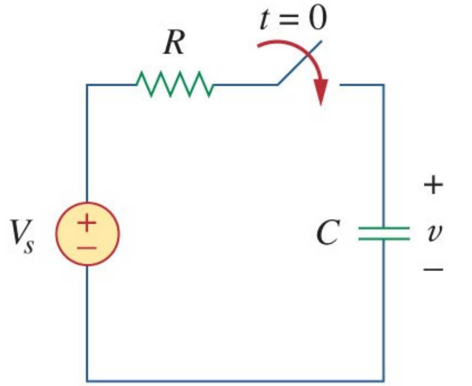
$$v(0^-) = v(0^+) = V_0$$

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$



- This is known as the complete response, or total response.

Complete response



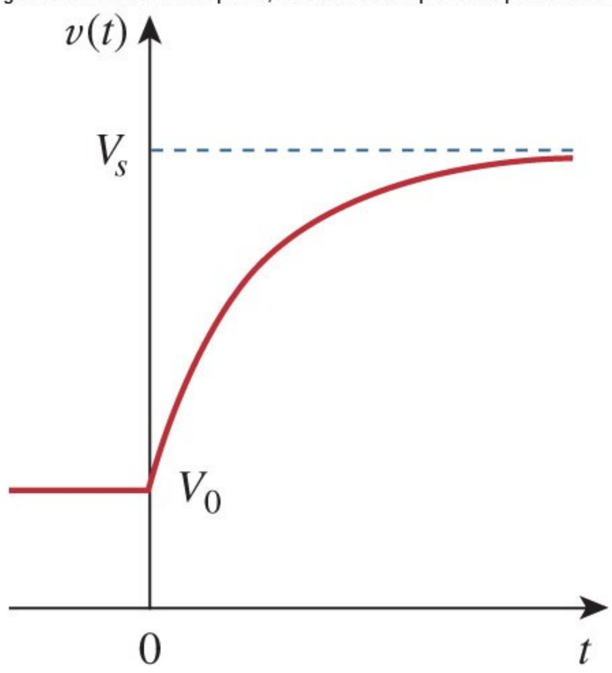
$$v(0^-) = v(0^+) = V_0$$

- The complete response

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

can be written as:

$$v = v_n + v_f$$



$$\underset{\text{}}{\text{Complete response}} = \underset{\text{stored energy}}{\text{natural response}} + \underset{\text{independent source}}{\text{forced response}}$$

or

$$v = v_n + v_f$$

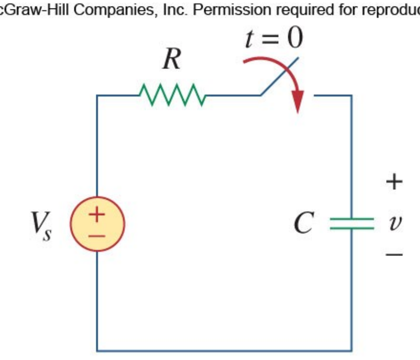
where

$$v_n = V_o e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

Another Perspective

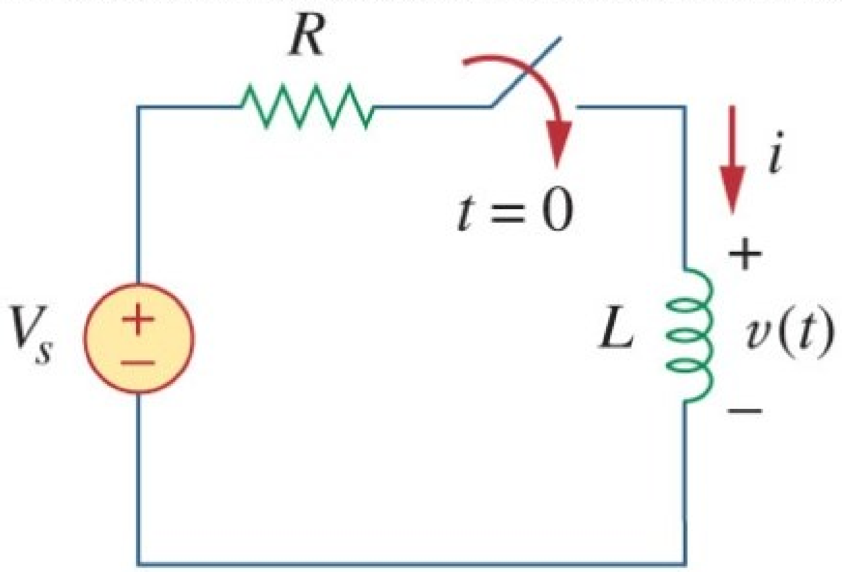


$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

- Another way to look at the response is to break it up into the transient response and the steady state response:

$$v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Step Response of the RL Circuit



$$i(0^+) = i(0^-) = I_0$$

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-t/\tau}$$

- Again one may break the response up into the transient response and the steady state response:

$$i(t) = \underbrace{i(\infty)}_{\text{steady } i_{ss}} + \underbrace{[i(0) - i(\infty)]e^{-t/\tau}}_{\text{transient } i_t}$$

General Procedure of Finding RC/RL Response with D.C. sources

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.

2. **Determine the initial value of the variable at T_0**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

$$i_L(T_0^+) = i_L(T_0^-) \quad \text{and} \quad v_c(T_0^+) = v_c(T_0^-)$$

3. **Determine the final value of the variable** (as $t \rightarrow \infty$)

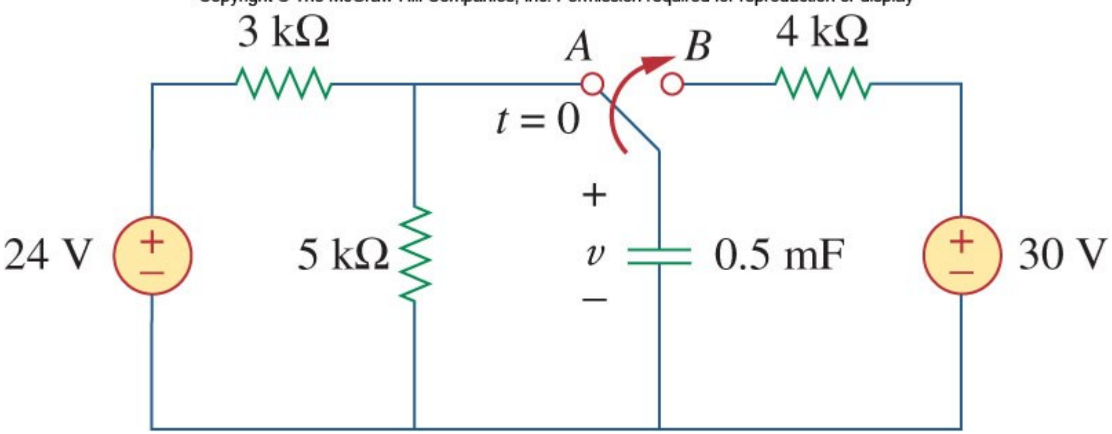
If needed, recall that an inductor behaves like a short circuit & that a capacitor behaves like an open circuit in steady state (e.g., $t \rightarrow \infty$).

4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

[Source: Berkeley]

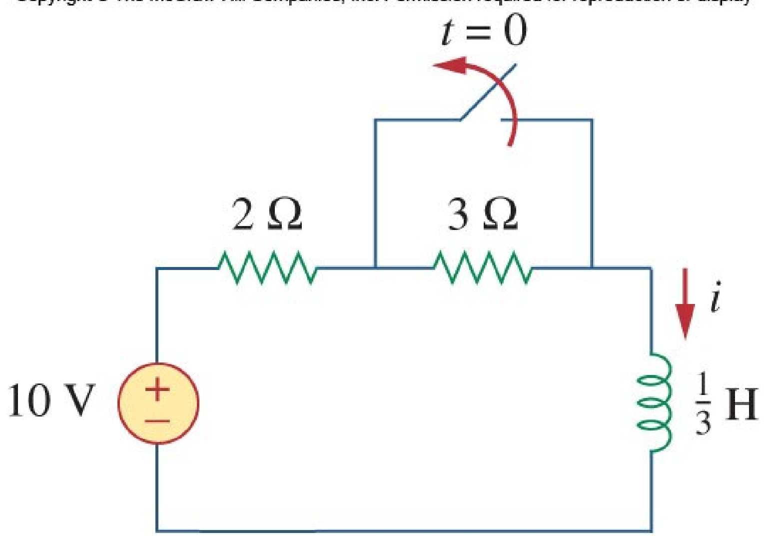
Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.



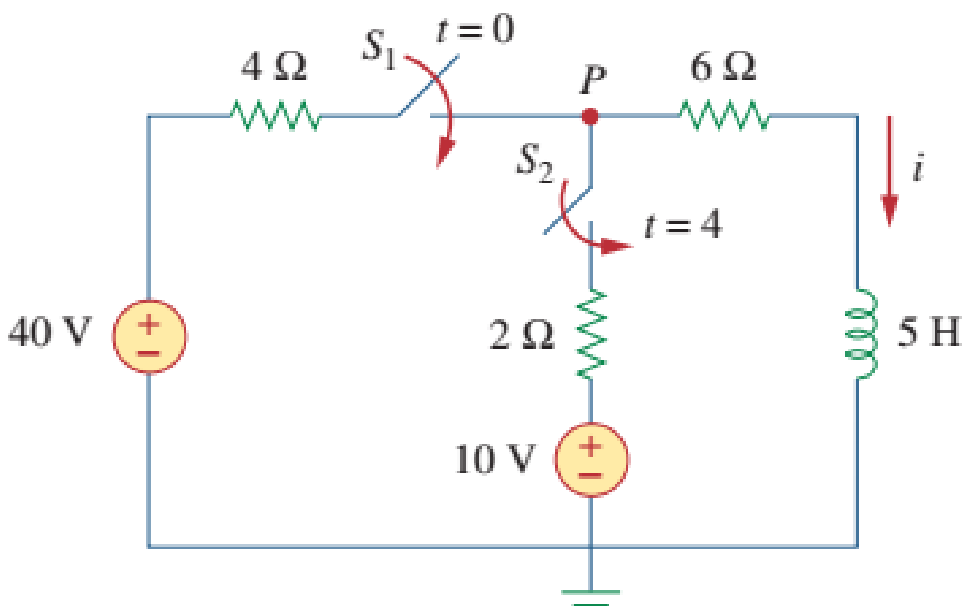
Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



Sequential switch

At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.

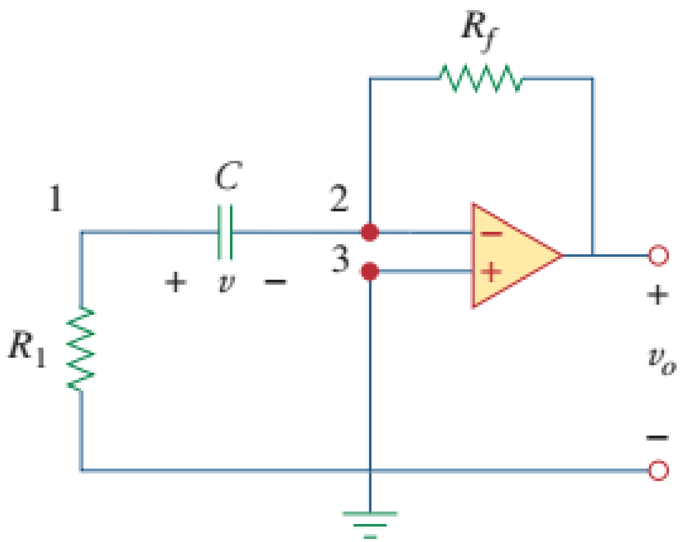


We need to consider the three time intervals $t \le 0$, $0 \le t \le 4$, and $t \ge 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

First order op-amp circuit

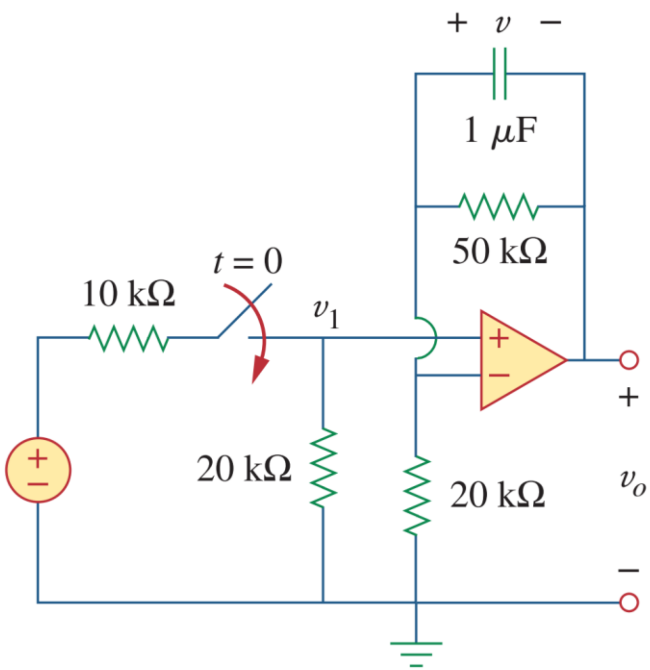
For the op amp circuit in Fig. 7.55(a), find v_o for $t > 0$, given that $v(0) = 3$ V. Let $R_f = 80\ \text{k}\Omega$, $R_1 = 20\ \text{k}\Omega$, and $C = 5\ \mu\text{F}$.



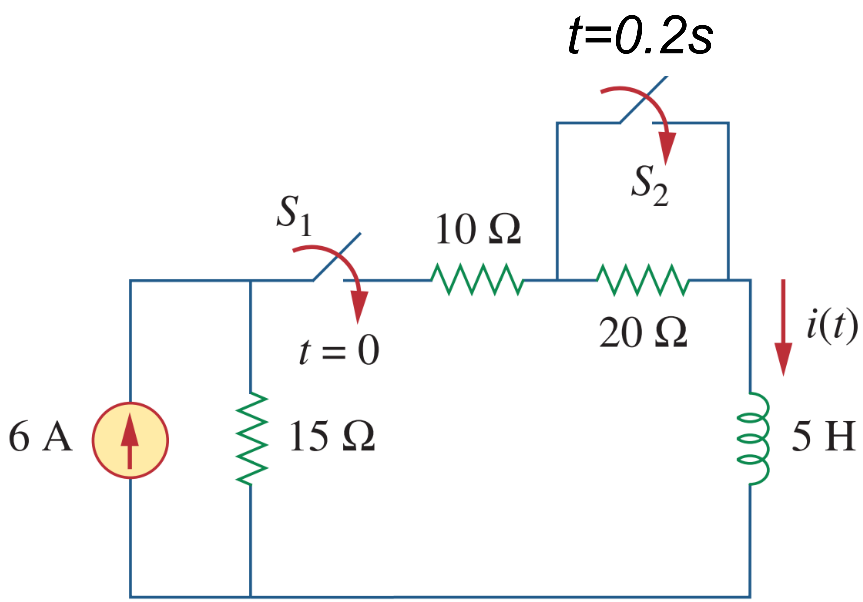
(a)

Non-DC Sources

$V_s = 3e^{-t}$



Practice



Practice