Lecture 14
-- Laplace Transform in Circuit Analysis

V-I relations of R,L,C

- R

$$U_R(s) = RI_R(s)$$

- C

$$V(s) = \frac{1}{sC} I(s) + \frac{V_0}{s}$$

$$I(s) = sCV(s) - CV_0$$

- L

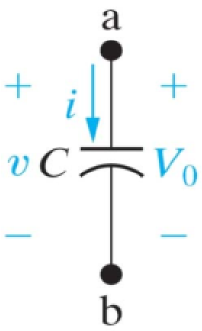
$$I(s) = \frac{1}{sL} V(s) + \frac{I_0}{s}$$

$$V(s) = sLI(s) - LI_0$$

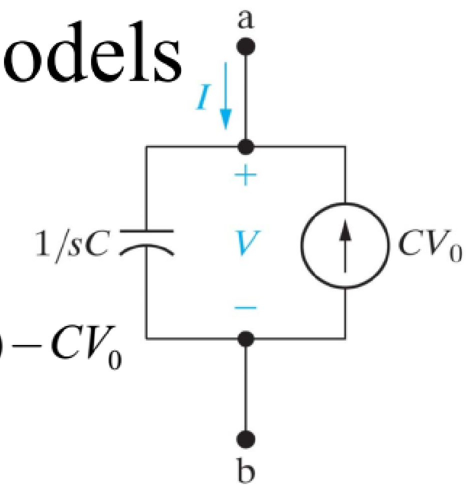
S-domain circuit models for a capacitor

s-Domain Circuit Models

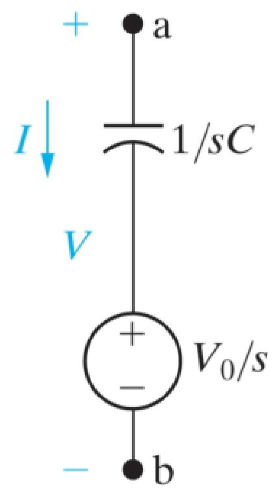
For a capacitor
(with initial conditions)



$$i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = sCV(s) - CV_0$$



$$V(s) = \frac{1}{sC}I(s) + \frac{V_0}{s}$$

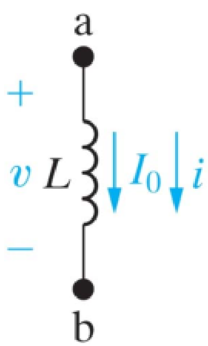


5

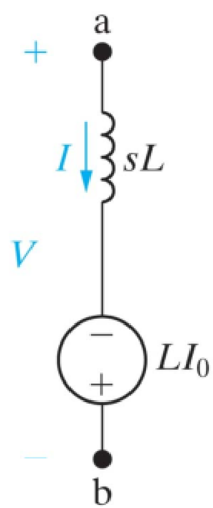
S-domain circuit models for an inductor

s-Domain Circuit Models

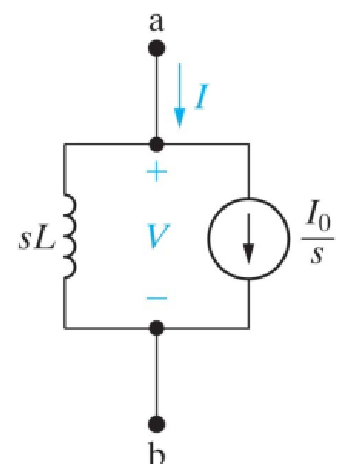
For an inductor
(with initial conditions)



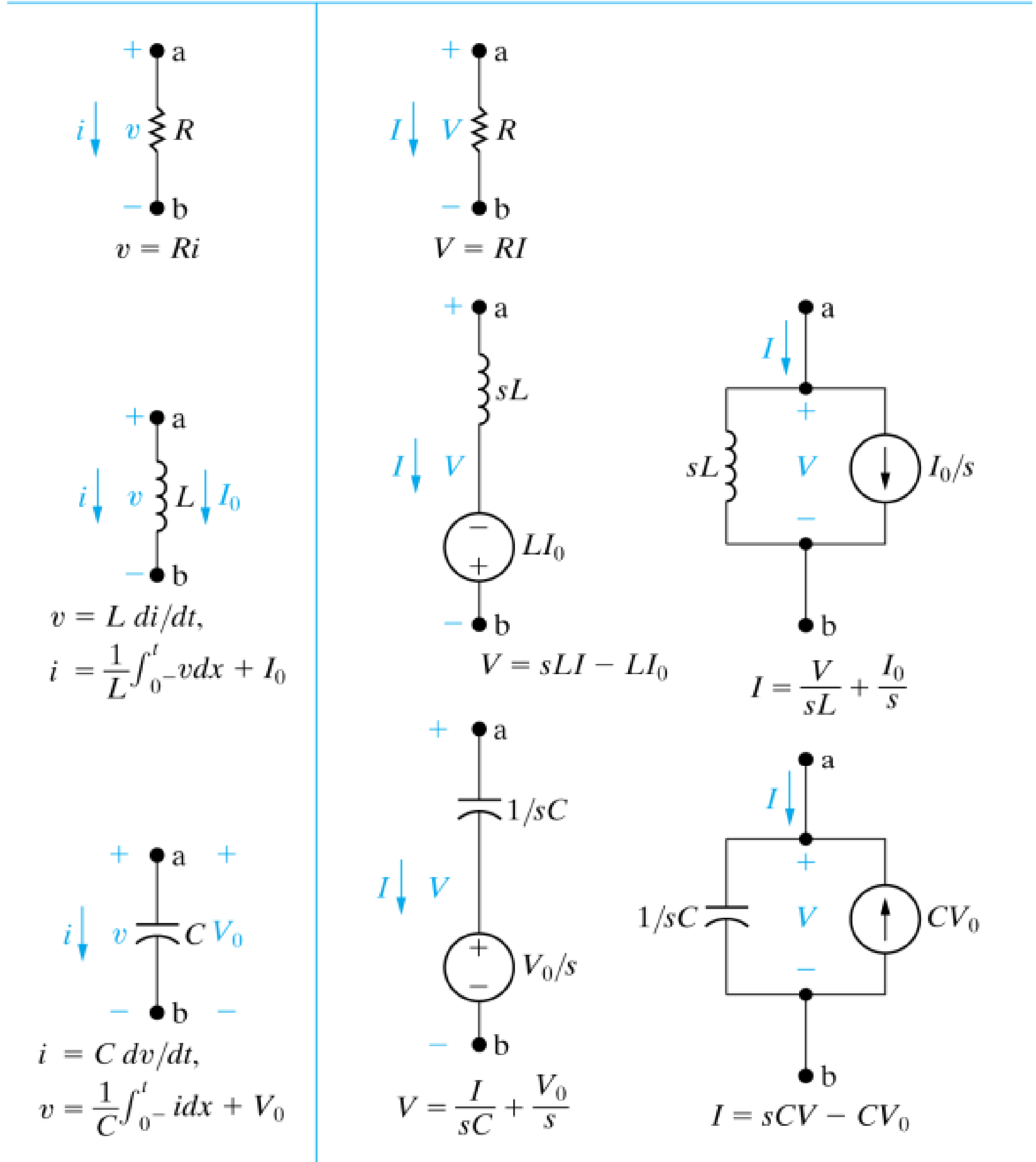
$$v(t) = L\frac{di(t)}{dt} \;\Rightarrow\; V(s) = sLI(s) - LI_0$$



$$I(s) = \frac{1}{sL}V(s) + \frac{I_0}{s}$$



Time domain	s-domain



D.C. sources and Dependent Sources

- The models for dependent sources are easy to develop, drawing from the simple fact that if the Laplace transform of *f(t)* is *F(s)*, then the Laplace transform of *af*(*t*) is *aF*(*s*) — the linearity property.

$$\mathcal{L}[av(t)] = aV(s)$$

$$\mathcal{L}[ai(t)] = aI(s)$$

Steps in Applying the Laplace transform

- **Transform the circuit from the time domain to the Laplace (*s*) domain, including initial conditions.**

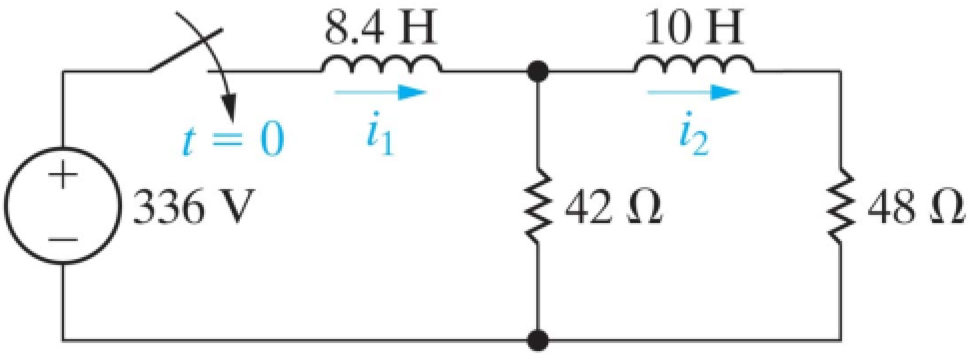
--*The elegance of using the Laplace transform in circuit analysis lies* in (1) transforming the differential equation into an *algebraic* equation; and (2) automatic inclusion of initial conditions in the transformation process, thus providing a complete (**transient and steady-state**) solution.

- **Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any other analysis technique with which we are familiar.**

- Take the inverse transform of the solution and thus obtain the solution in the time domain.

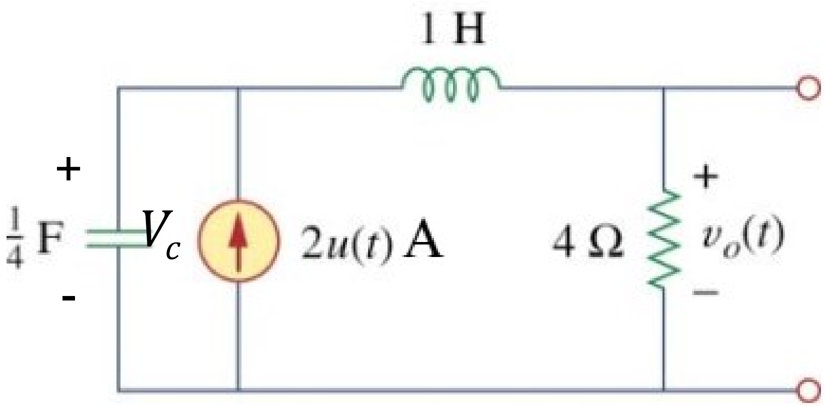
Example 1

Assuming no initial energy storage, find $i_1(t)$ and $i_2(t)$ for $t > 0$.



Example 2

Given $V_c(t = 0) = 2V$, determine $v_o(t)$ *for* $t>0$

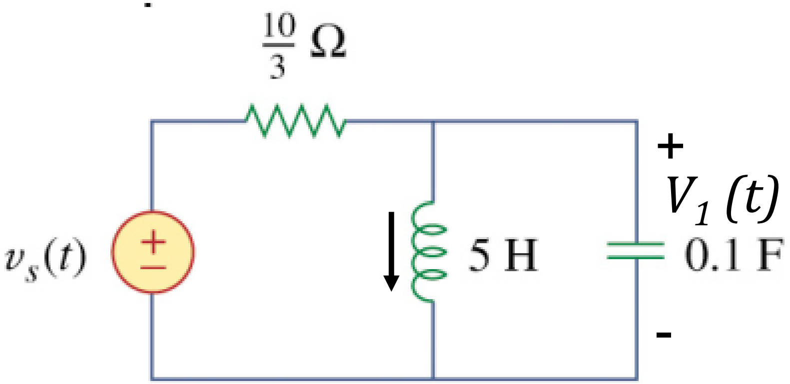


Example 3

- Find (1) the voltage across the capacitor

  (2) current through the inductor

  assuming that $v_s(t) = 10u(t)$ V, and given that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.

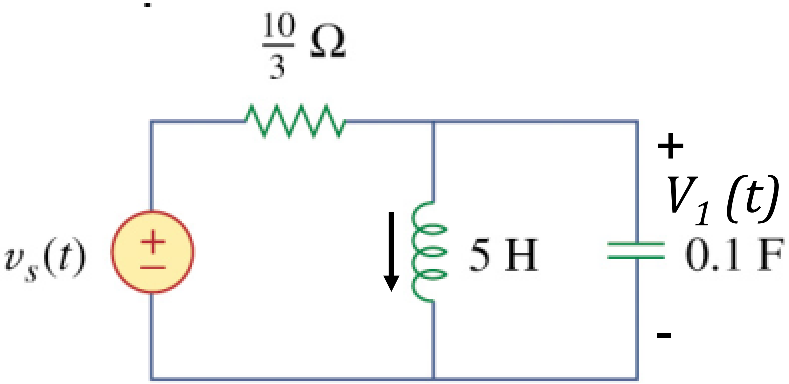


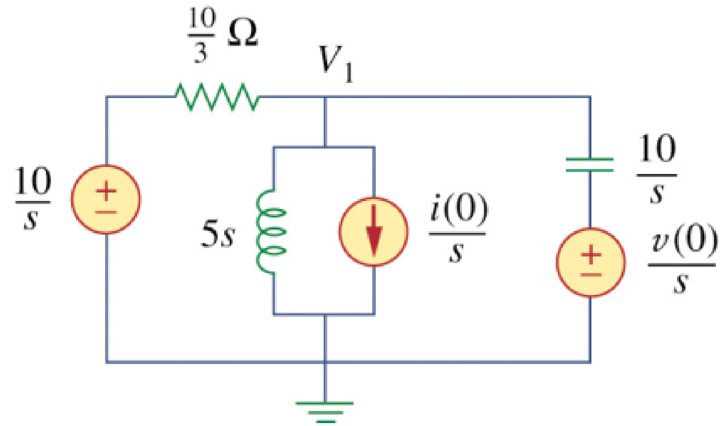
Example 3

- Find (1) the voltage across the capacitor

  (2) current through the inductor

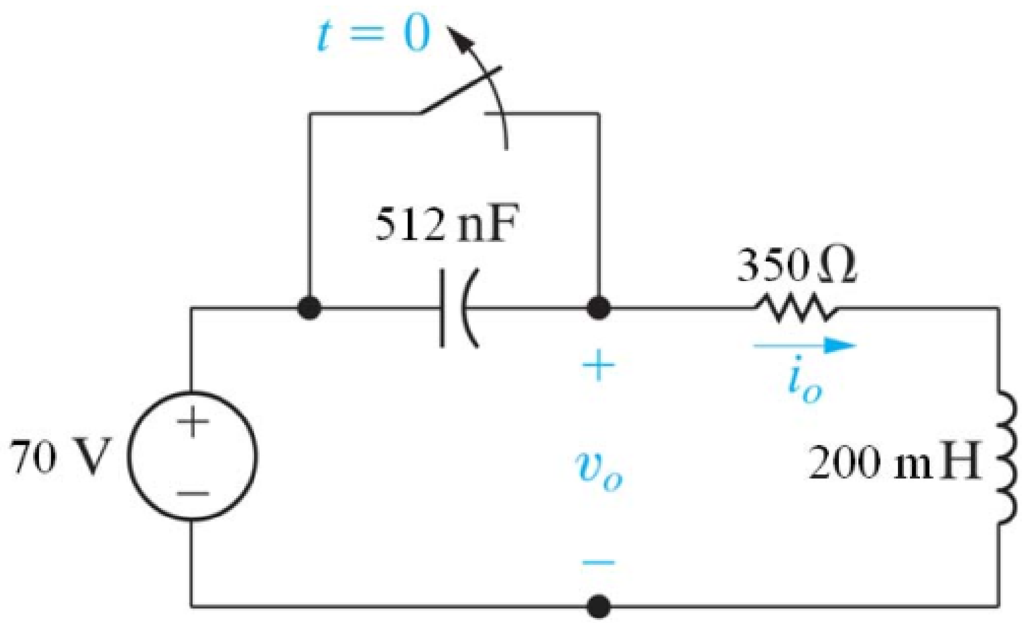
  assuming that $v_s(t) = 10u(t)$ V, and assume that at $t = 0$, *-1 A* flows through the inductor and +5 V is across the capacitor.





Example 4

- Find $v_o(t)$ *for* $t>0$



$$V_0(s) = \frac{70s - 268{,}125}{s^2 + 1750s + 9{,}765{,}625} = \frac{K_1}{(s + 875 - j3000)} + \frac{K_2}{(s + 875 + j3000)}$$

$$K_1 = \left.\frac{70s - 268{,}125}{(s + 875 + j3000)}\right|_{s=-875+j3000} = \frac{70(-875 + j3000) - 268{,}125}{[(-875 + j3000) + 875 + j3000]} = 65.1\angle 57.48^\circ$$

$$K_2 = \left.\frac{70s - 268{,}125}{(s + 875 - j3000)}\right|_{s=-875-j3000} = \frac{70(-875 - j3000) - 268{,}125}{[(-875 - j3000) + 875 + -j3000]} = 65.1\angle -57.48^\circ$$

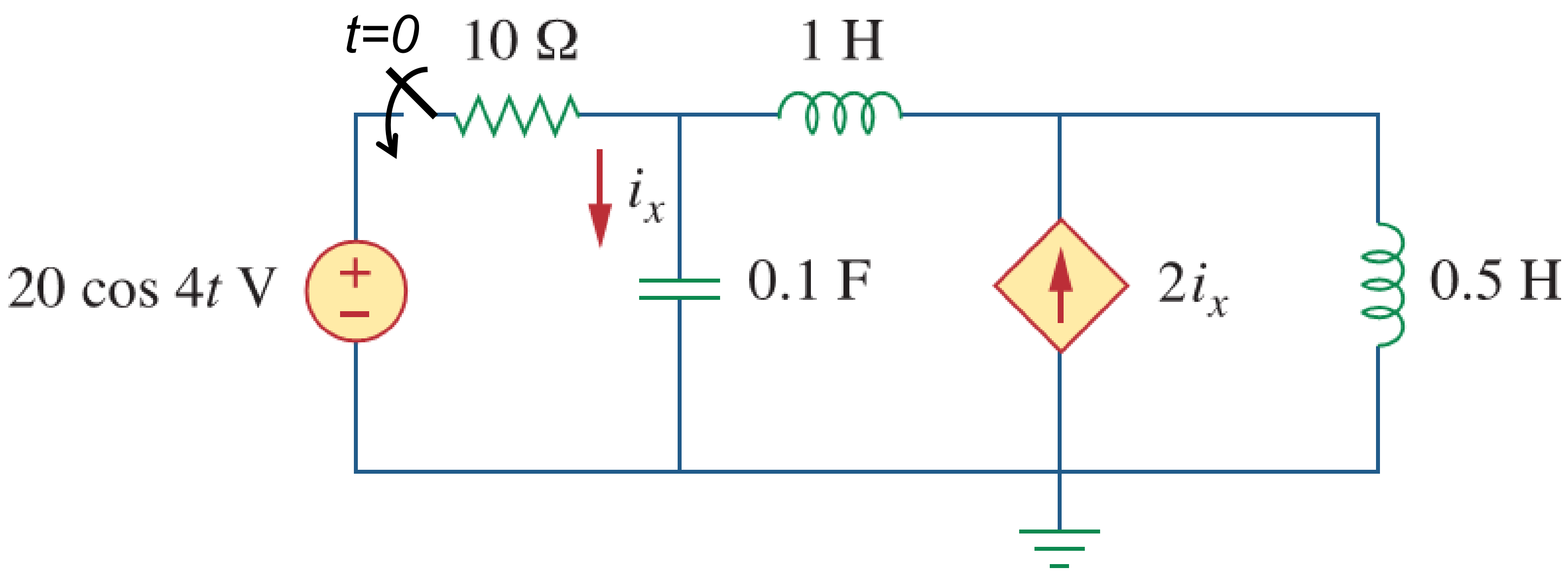
$$V_0(s) = \frac{65.1\angle 57.48^\circ}{(s + 875 - j3000)} + \frac{65.1\angle -57.48^\circ}{(s + 875 + j3000)}$$

$$v_0(t) = 2(65.1)e^{-875t}\cos(3000t + 57.48^\circ) = 130.2e^{-875t}\cos(3000t + 57.48^\circ)u(t)\ \mathrm{V}$$

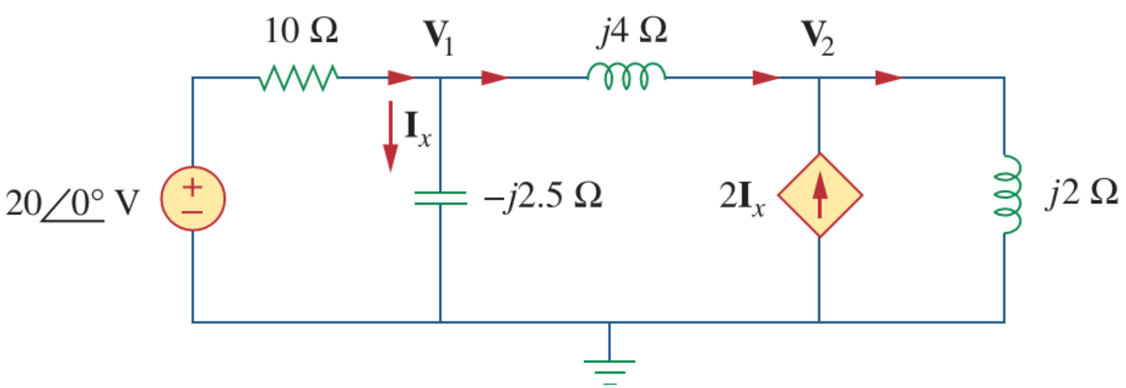
Example 5

- Example---Find i_x(s.s.) assuming no initial energy stored

Using phasor method and Laplace transform method

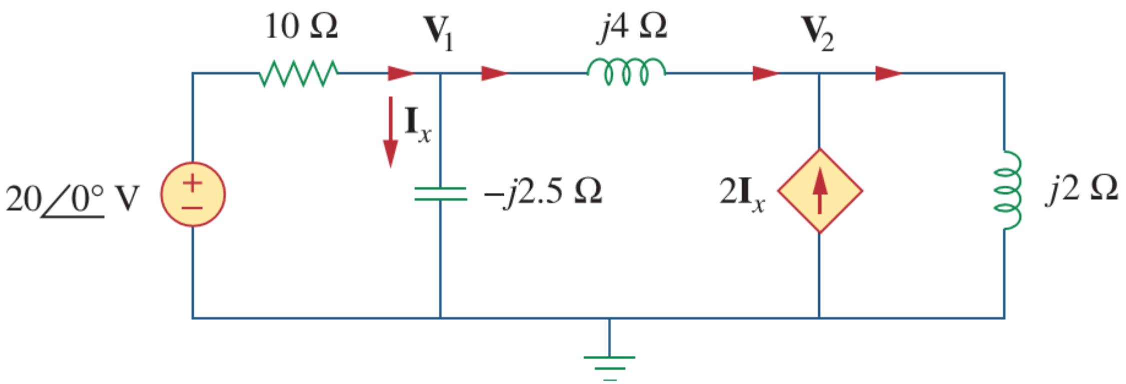


Phasor method



$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$



$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$i_x = 7.59 \cos(4t + 108.4°) \text{ A}$$

Laplace transform method to find i_x