



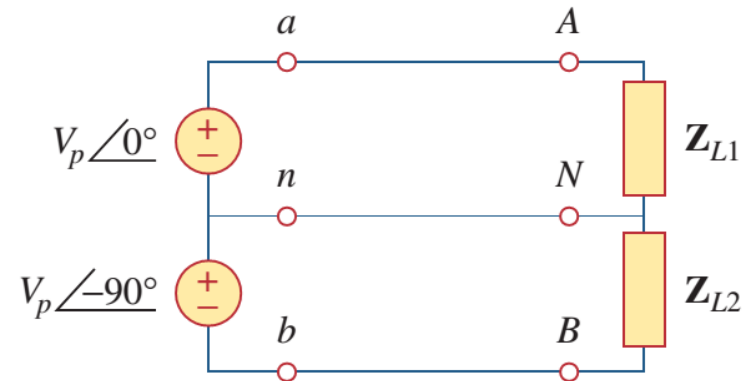
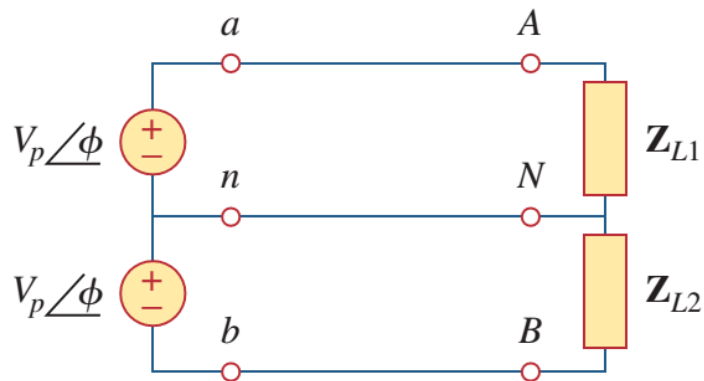
Lecture 10

- Three-Phase Circuits



Single phase vs. Polyphase

- Single-phase power supply
 - For example, two 120V sources with the same phase are connected in series.
 - This allows for appliances to use either 120 or 240V
- Circuits that operate with multiple sources, at the same frequency but *at different phases* are called polyphase.





Outline--Three-Phase Circuits

- Balanced Three-Phase System
 - Balanced sources
 - Balanced loads
- Circuit analysis
 - Phase voltage/current
 - Line voltage/current

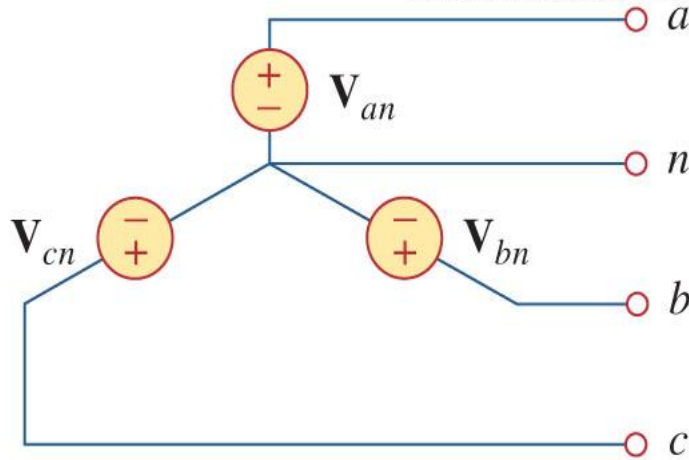


Balanced Three-Phase Sources

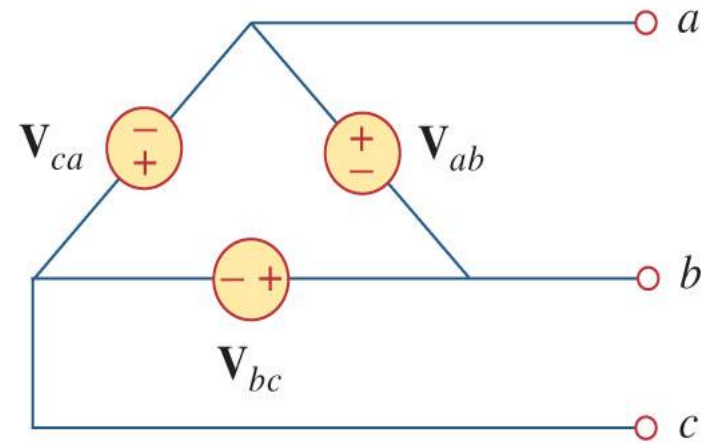
Connecting the Sources

- Three phase voltage sources can be connected by either four or three wire configurations.
 - Four-wire system accomplished using a Y(Wye) connected source.
 - Three-wire configuration accomplished by Delta connected source.

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(a)



(b)

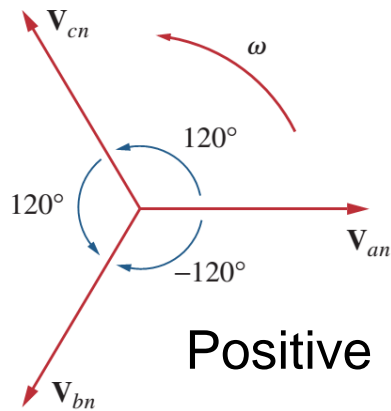
$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

Balanced Three-Phase Sources

- **Balanced phase voltage** are equal in magnitude and are **out of phase with each other by 120deg**
- It's easy to know $V_{an} + V_{bn} + V_{cn} = 0$
- Two sequences for the phases:

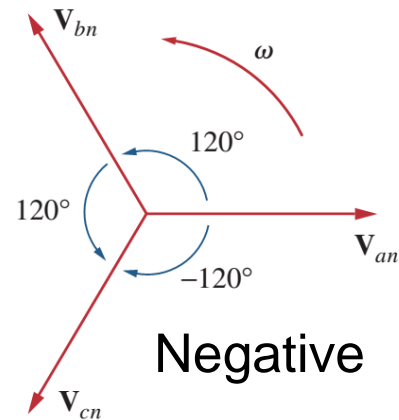


Positive

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$



Negative

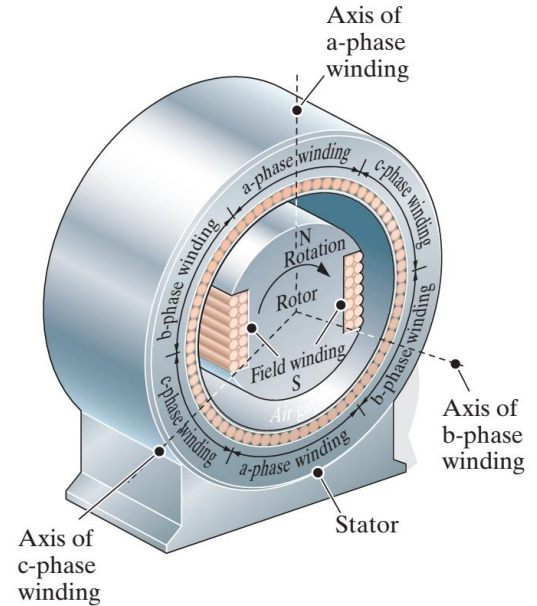
$$V_{an} = V_p \angle 0^\circ$$

$$V_{cn} = V_p \angle -120^\circ$$

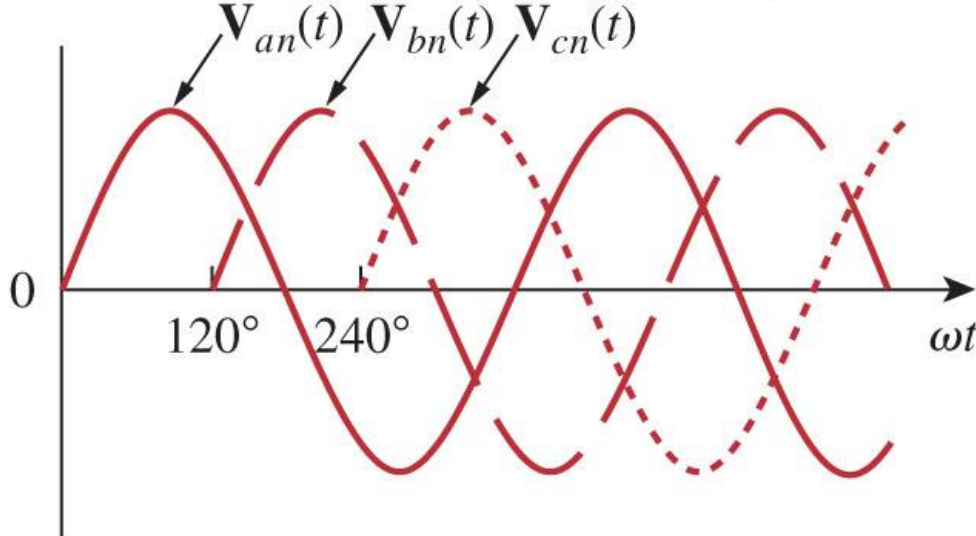
$$V_{bn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

Three-Phase Sources

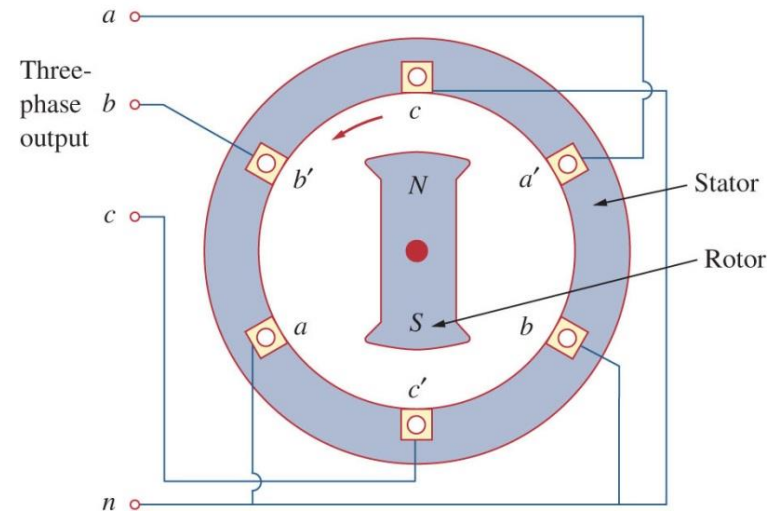
- Three phase voltages are typically produced by a three-phase AC generator.
- The output voltages look like below.



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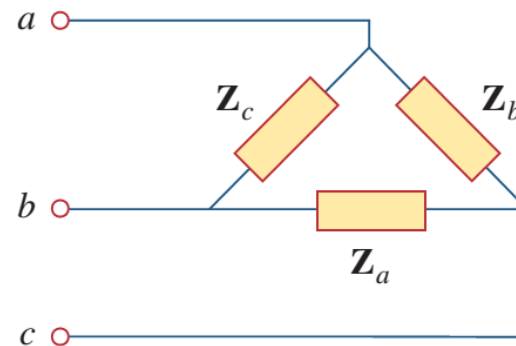
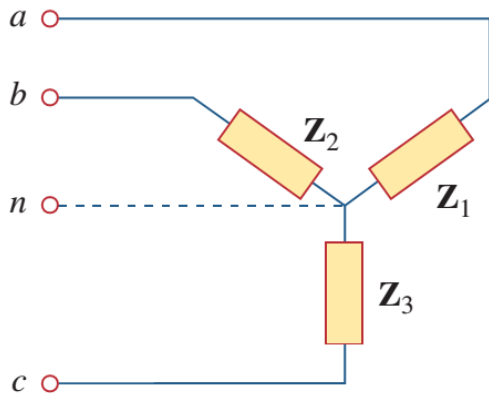


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Balanced Loads

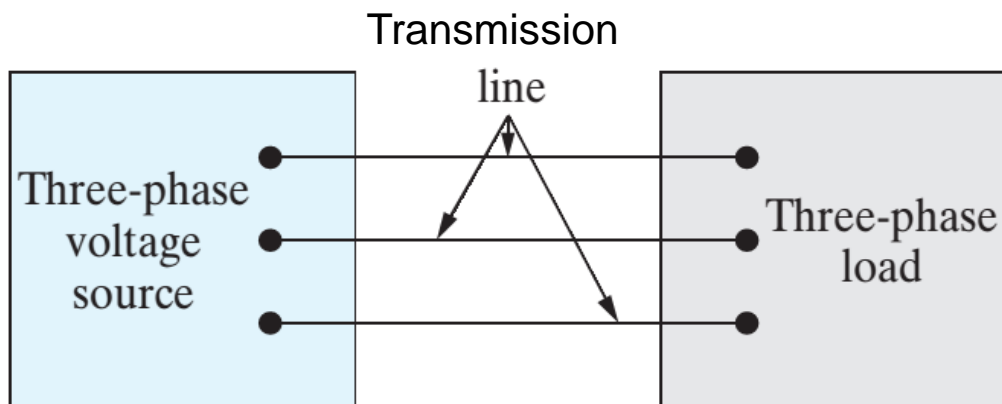
- A balanced load means the same impedance for each load.
-- Impedance are equal in magnitude and share the same impedance angle
- They may also be connected in either Delta or wye
 - For a balanced wye connected load: $Z_1 = Z_2 = Z_3 = Z_Y$
 - For a balanced delta connected load: $Z_a = Z_b = Z_c = Z_\Delta$



- The load impedance per phase for the above configurations can be **interchanged**.



Source-Load configurations

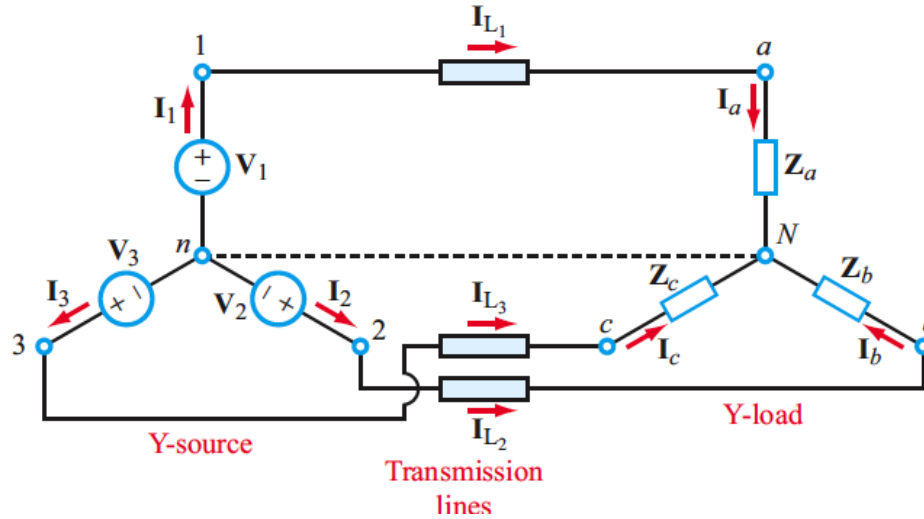


Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ



Source-Load Configurations

Y-Y



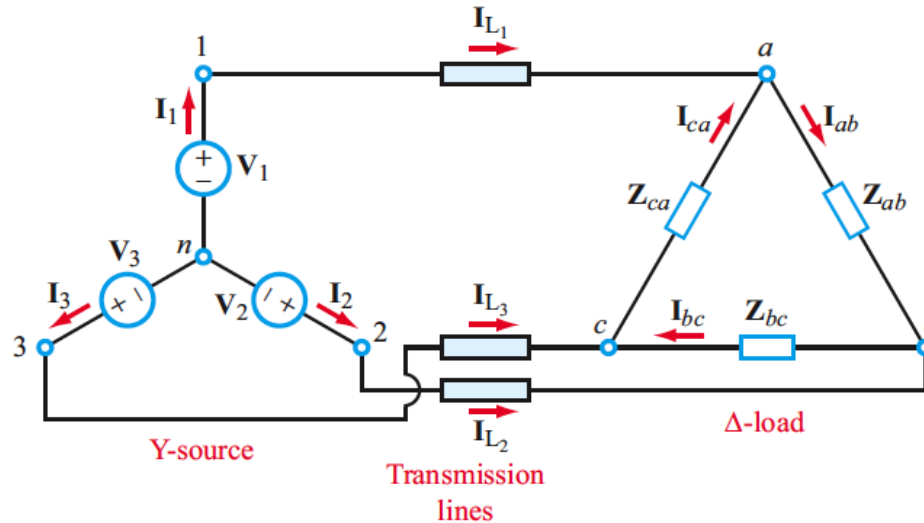
Load Phase Currents

I_a, I_b, I_c
(same as line currents
 $I_{L1}, I_{L2},$ and I_{L3})

Load Phase Voltages

V_{aN}, V_{bN}, V_{cN}

Y-Delta



Load Phase Currents

I_{ab}, I_{bc}, I_{ca}

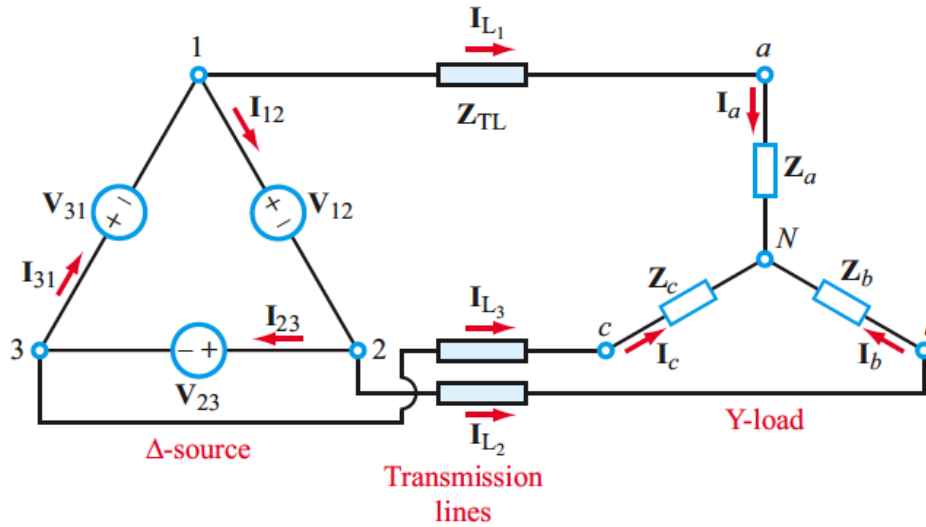
Load Phase Voltages

V_{ab}, V_{bc}, V_{ca}



Source-Load Configurations

Delta-Y



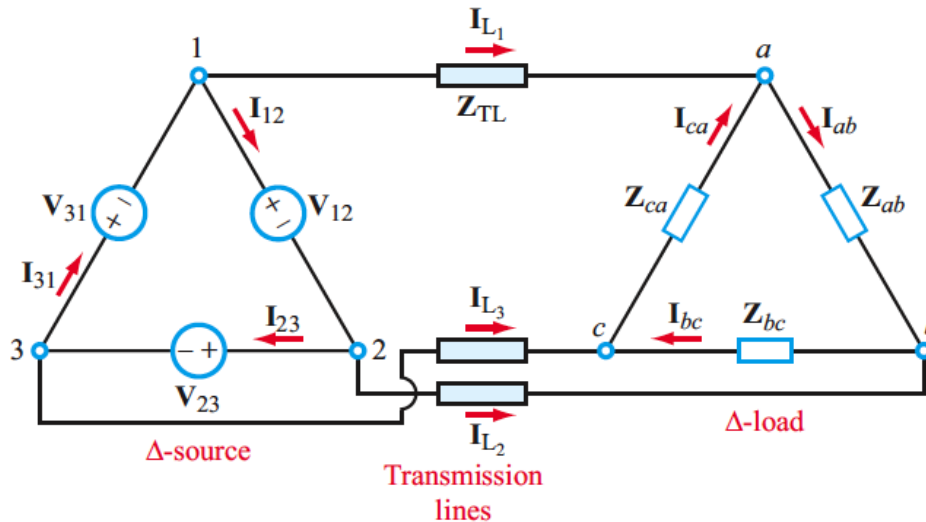
Load Phase Currents

I_a, I_b, I_c
(same as line currents
 $I_{L1}, I_{L2},$ and I_{L3})

Load Phase Voltages

V_{aN}, V_{bN}, V_{cN}

Delta-Delta



Load Phase Currents

I_{ab}, I_{bc}, I_{ca}

Load Phase Voltages

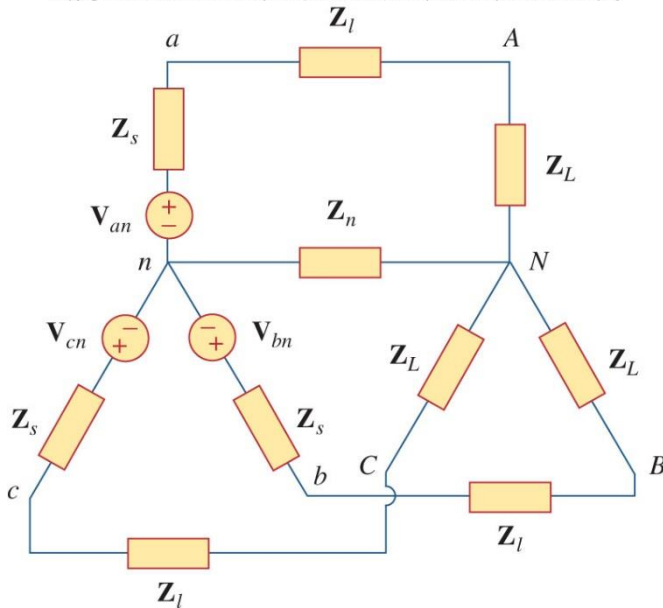
V_{ab}, V_{bc}, V_{ca}
(same as source voltages
if Z_{TL} is negligible)

Balanced Y-Y connection

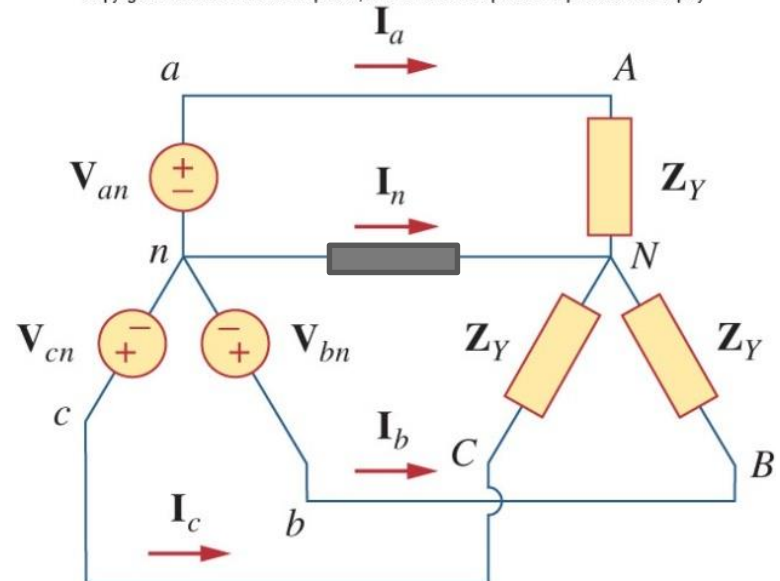
- The load impedances Z_Y will be assumed to be balanced.
 - This can be the source Z_S , line Z_l and load Z_L together.

$$Z_Y = Z_S + Z_\ell + Z_L$$

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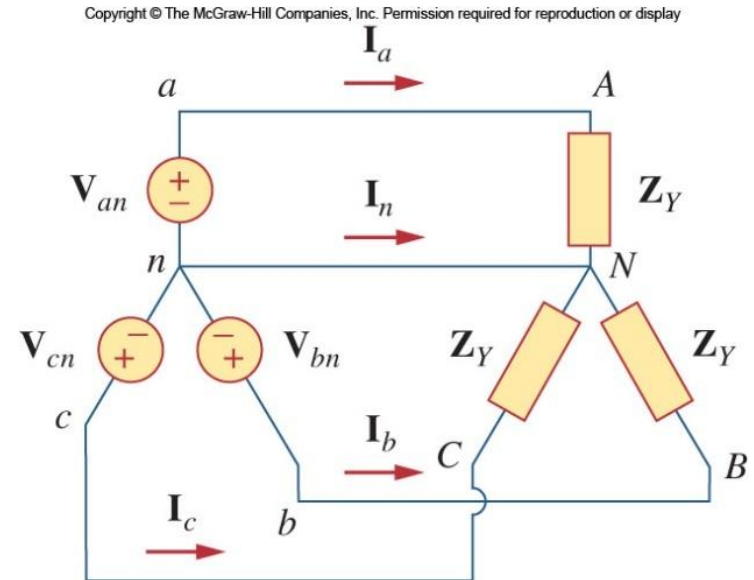
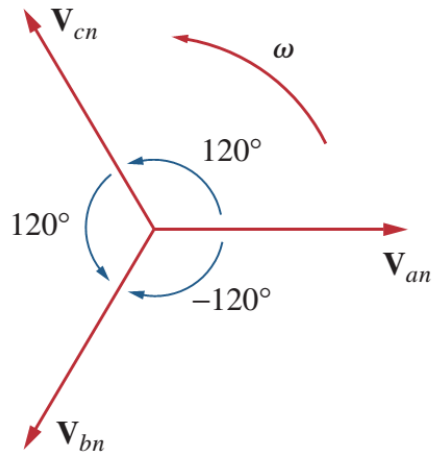
Phase Voltage & Line-to-Line Voltage

- Use the positive sequence:

Phase Voltage

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ \quad V_{cn} = V_p \angle +120^\circ$$

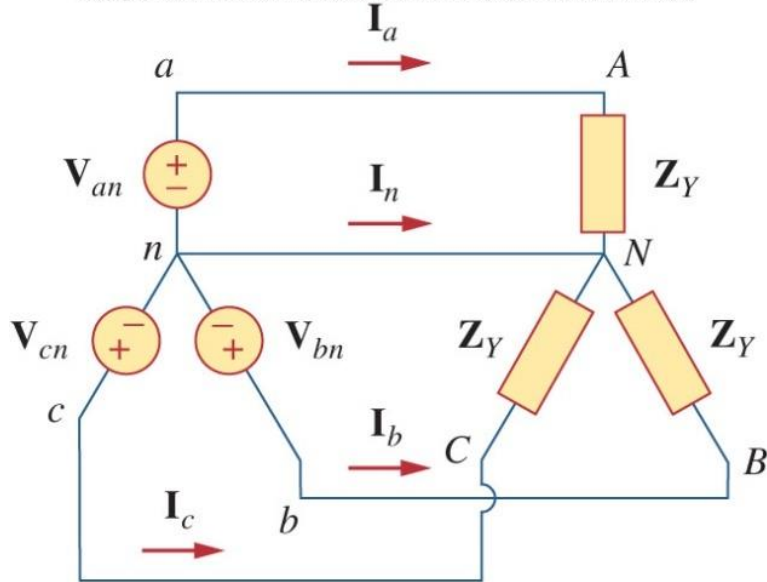


- The line to line voltages (or just line voltages in short):**



Line Currents

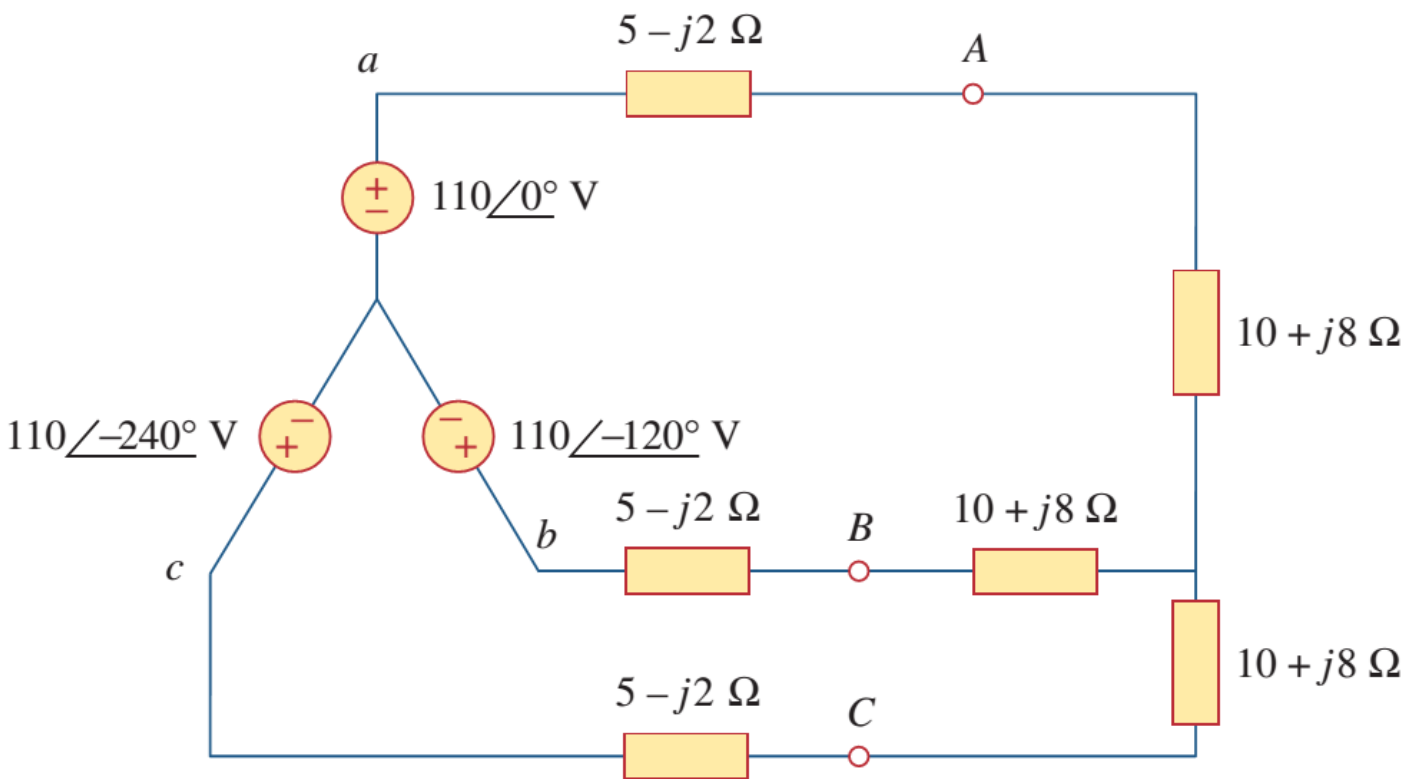
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Example

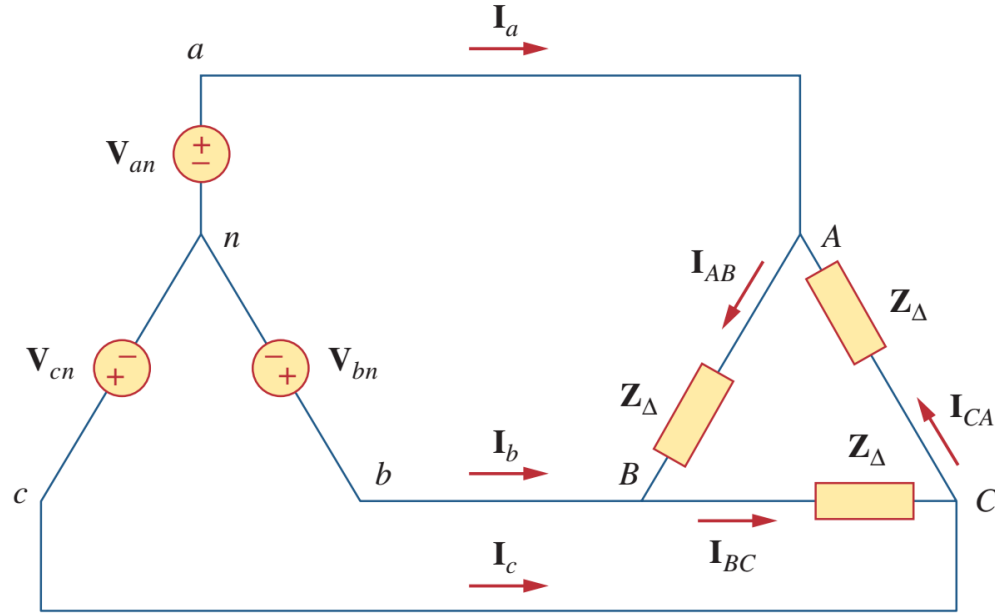
- Calculate the line currents.





Wye- Δ

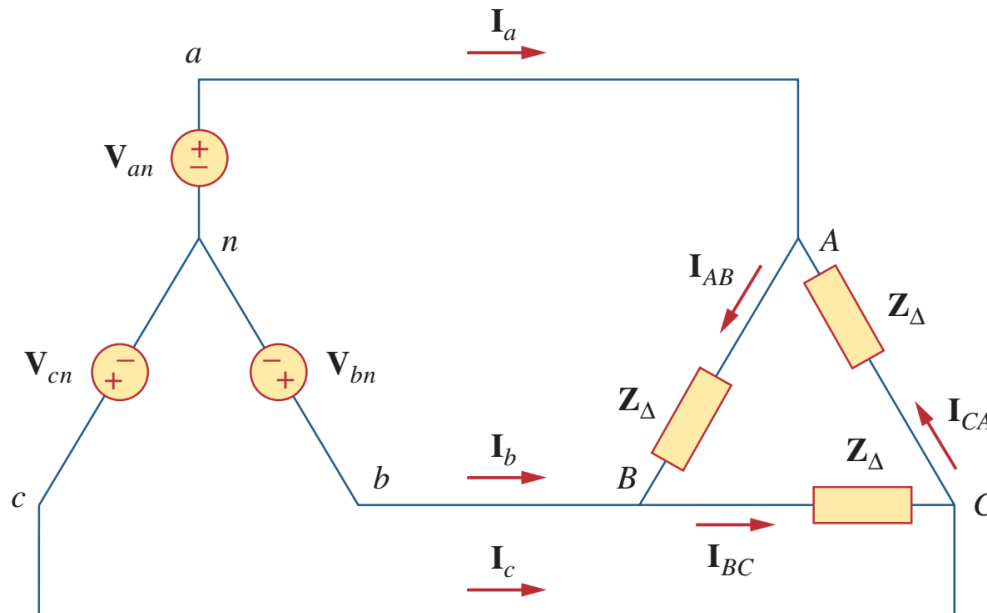
Find load phase current
and line current





Wye- Δ

Find load phase current and line current



Assume positive sequence:

$$\begin{aligned} \mathbf{V}_{an} &= V_p \angle 0^\circ \\ \mathbf{V}_{bn} &= V_p \angle -120^\circ, & \mathbf{V}_{cn} &= V_p \angle +120^\circ \end{aligned}$$

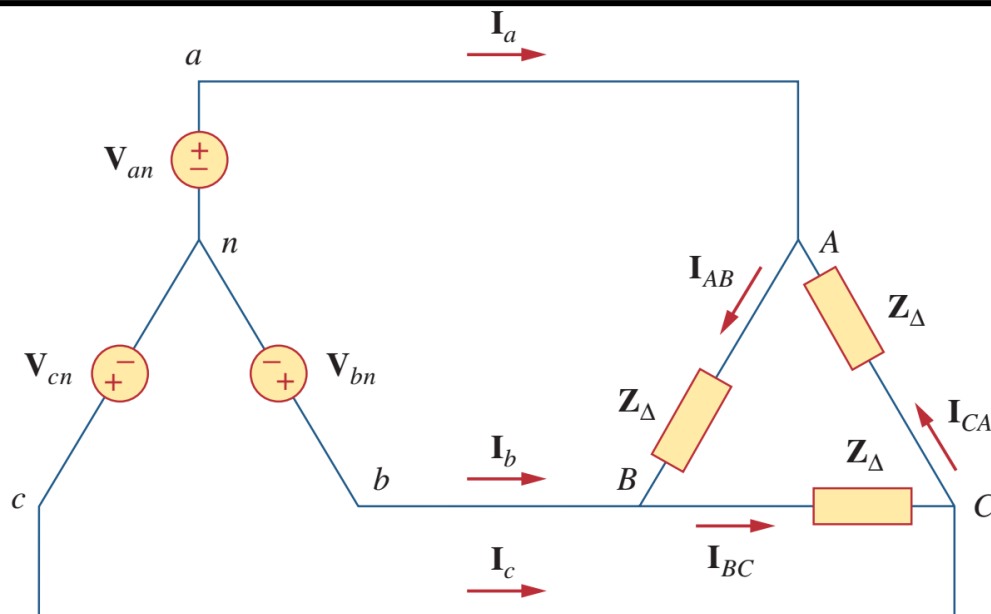
$$\mathbf{V}_{ab} = \sqrt{3}V_p \angle 30^\circ = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \sqrt{3}V_p \angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p \angle -150^\circ = \mathbf{V}_{CA}$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \quad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \quad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$



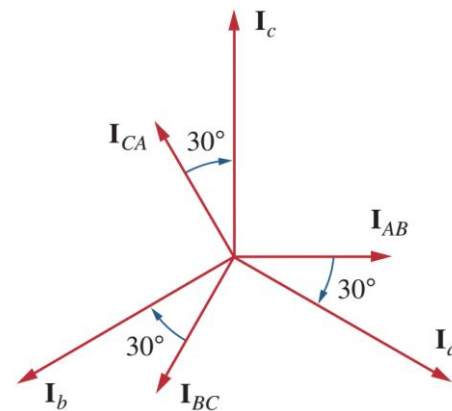
Wye- Δ



$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \quad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \quad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

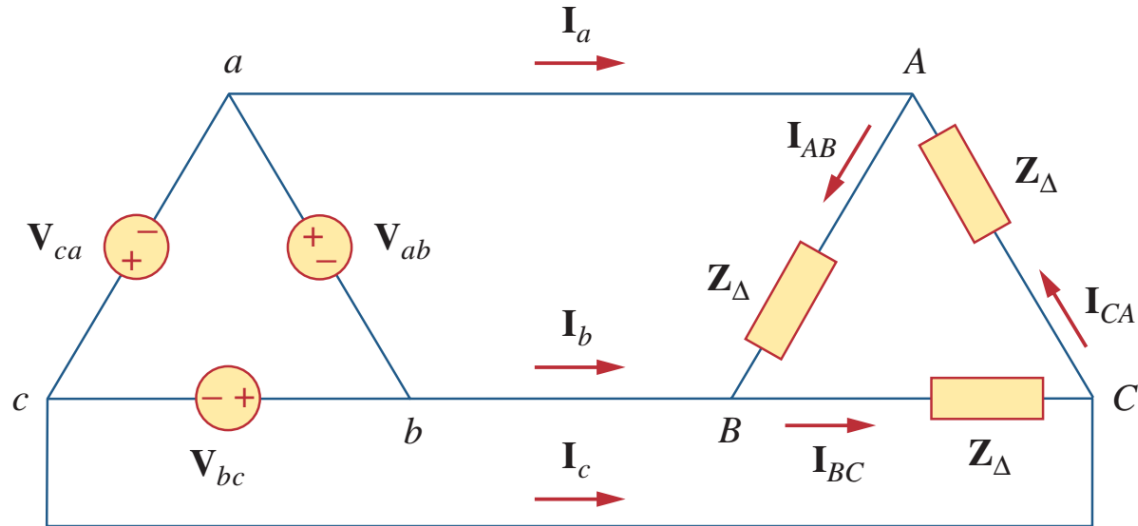
Since $\mathbf{I}_{CA} = \mathbf{I}_{AB} \angle -240^\circ$,

$$\begin{aligned} \mathbf{I}_a &= \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1 \angle -240^\circ) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB} \sqrt{3} \angle -30^\circ \end{aligned}$$





• Δ - Δ





Δ -Wye

